MATEMATIKA 2020/2021

1.

$$A=\begin{pmatrix} 2 & 2 & -1 \\ 0 & 1 & 0 \\ 5 & 3 & 4 \end{pmatrix}$$

a) vlastní čísla

b) vlastní vektory

a) $\det(A-\lambda E)=0$

$$\det\begin{vmatrix} 2-\lambda & 2 & -1 \\ 0 & 1-\lambda & 0 \\ 5 & 3 & 4-\lambda \end{vmatrix}=0$$

$(2-\lambda)(1-\lambda)(4-\lambda)+0+0-(-1)(1-\lambda)5-0-0=0$

$(2-\lambda)(1-\lambda)(4-\lambda)+5(1-\lambda)=0$

$(4-\lambda)(2-2\lambda-\lambda+\lambda^2)+5-5\lambda=0$

$\cancel{8}\ (1-\lambda)\left[8-2\lambda-4\lambda+\lambda^2+5\right]=0$

$\lambda_1=1$

$\lambda^2-6\lambda+13=0$

$D=36-4\cdot 13<0$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 5 & 3 & 3 \end{pmatrix}$$

$u_3=v$

$\downarrow$

$u_3=7=v$

$u_1+2u_2-v=0 \quad |\cdot(-5)$

$5u_1+3u_2+3v=0$

$-7u_2+8v=0$

$u_2=\frac{8}{7}v$

$u_1+2\frac{8}{7}v-v=0$

$u_1+\frac{9}{7}v=0$

$u_1=-\frac{9}{7}v$

$u=\begin{pmatrix} -9 \\ 8 \\ 7 \end{pmatrix}v \qquad v\neq 0;\ v\in R$

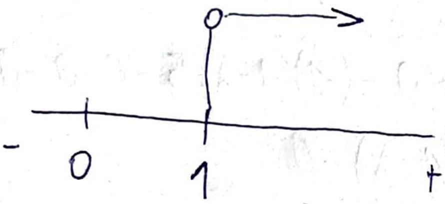
2. $f(x) = \ln(x-1) - \sqrt{x-1}$

a) $D(f)$

b) rovnice tečny $[x_0; f(x_0)]$; $x_0 = 2$

hodnota $f(x)$ pro $x = \frac{5}{2}$

$\underline{x-1>0} \wedge x-1 \geq 0$



$D(f) = (1, +\infty)$

$f'(x) = \frac{1}{x-1} - \left[\frac{1}{2}(x-1)^{-\frac{1}{2}} \cdot 1\right] =$

$= \frac{1}{x-1} - \frac{1}{2\sqrt{x-1}}$

$y = f(x_0) + f'(x_0)(x - x_0)$

$f'(2) = 1 - \frac{1}{2}$

$f(2) = 0 - 1 = -1$

$\underline{y = -1 + \frac{1}{2}(x-2)}$

$x = \frac{5}{2}$

$y\left(\frac{5}{2}\right) = -1 + \frac{1}{2}\left(\frac{5}{2} - 2\right) =$

$= -1 + \frac{1}{2} \cdot \frac{1}{2} = \underline{-\frac{3}{4}}$

$f(x,y) = 3\sqrt{x} - y$

$y = x + 1 \quad x \in \langle 0, 4 \rangle$

$f(x) = 3\sqrt{x} - x - 1$

$f'(x) = 3 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} - 1 = \frac{3}{2}\frac{1}{\sqrt{x}} - 1$

$\frac{3}{2}\frac{1}{\sqrt{x}} - 1 = 0$

$\frac{3}{2}\frac{1}{\sqrt{x}} = 1$

$x = \frac{9}{4}$

$y = \frac{9}{4} + 1 = \frac{13}{4}$

$y = 1$ pro $x = 0$

max: $f\left(\frac{9}{4}; \frac{13}{4}\right) = 3\frac{\sqrt{9}}{\sqrt{4}} - \frac{13}{4} = \frac{9}{2} - \frac{13}{4} = \underline{\frac{5}{4}}$

min: $f(0, 1) = \underline{-1}$

$f(4, 5) = 3\sqrt{4} - 5 = 1$

③ $\ddot{x} - 3\dot{x} - 4x = 6e^{-2t}$

a) $\lambda^2 - 3\lambda - 4 = 0$

$D = 9 - 4(-4) = 25$

$\lambda_{1,2} = \dfrac{3 \pm 5}{2} = \begin{cases} 4 \\ -1 \end{cases}$

$x_H = C_1 \cdot e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

Fundamentální systém: $\{ \varphi_1(t) = e^{4t} ; \varphi_2(t) = e^{-t} \}$

Obecné řešení:

$x(t) = C_1 e^{4t} + C_2 e^{-t} \; ; \; t \in (-\infty, +\infty)$

b) odhad $x_p = A \cdot e^{-2t}$

$\dot{x}_p = A \cdot (-2) \cdot e^{-2t}$

$\ddot{x}_p = A\, 4 e^{-2t}$

$x_p = e^{-2t}$

$4Ae^{-2t} + 6Ae^{-2t} - 4Ae^{-2t} = 6e^{-2t}$

$4A + 6A - 4A = 6$

$6A = 6$

$A = 1$

– ani jedno λ se neshoduje s koeficientem v $e^{(-2)}$, bude tedy $x_p = A \cdot e^{-2t}$

– pokud je λ shodné s exponentem v e^{0}, pak $x_p = A \cdot t\, e^{-0t}$

Obecné řešení:

$x(t) = C_1 e^{4t} + C_2 e^{-t} + e^{-2t} \; ; \; t \in (-\infty, \infty)$

c) $x(0) = 1$
$\dot{x}(0) = 3$

$x(t) = C_1 e^{4t} + C_2 e^{-t} + e^{-2t}$

$\dot{x}(t) = 4C_1 e^{4t} - C_2 e^{-t} + (-2) e^{-2t}$

$1 = C_1 + C_2 + 1$

$3 = 4C_1 - C_2 - 2$

$C_1 = 1 \; ; \; C_2 = -1$

$x(t) = e^{4t} - e^{-t} + e^{-2t} \; ; \; t \in (-\infty, +\infty)$

2019/2020

① $f(x) = \ln(2x-1) - \frac{x}{2}$

a) $D(f), f', T_2(x), x_0 = 1, f\left(\frac{5}{4}\right)$

$2x-1 > 0 \quad \wedge \dots$

$x > \frac{1}{2} \qquad D(f) = \left(\frac{1}{2}; +\infty\right)$

$$f'(x) = \frac{1\cdot 2}{2x-1} - \frac{1}{2} = \frac{2}{2x-1} - \frac{1}{2}$$

$$f'(x) = \frac{4-2x+1}{4x-2} = \frac{5-2x}{4x-2}$$

$$f''(x) = \frac{0-2\cdot 2}{(2x-1)^2} = -\frac{4}{(2x-1)^2}$$

$$f'''(\xi) = \frac{16}{(2\xi-1)^3} \leftarrow f'''(x) = -\frac{0-4\cdot 2(2x-1)\cdot 2}{(2x-1)^4} = \frac{16}{(2x-1)^3}$$

$x_0 = 1$

$$T_2(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$$

$$T_2(x) = -\frac{1}{2} + \frac{3}{2}(x-1) + \frac{-4}{2}(x-1)^2 = -\frac{1}{2} + \frac{3}{2}(x-1) - 2(x-1)^2$$

↗ rastie v okolí $x_0 = 1$

$$T_2\left(\frac{5}{4}\right) = f\left(\frac{5}{4}\right) = -\frac{1}{2} + \frac{3}{2}\left(\frac{5}{4}-1\right) - 2\left(\frac{5}{4}-1\right)^2 = -\frac{1}{2} + \frac{3}{2}\cdot\frac{1}{4} - 2\cdot\cancel{\frac{25}{16}}\frac{1}{16} =$$

$$= -\frac{1}{2} + \frac{3}{8} - \frac{1}{8} = \frac{-4+3-1}{8} = -\frac{2}{8} = \underline{\underline{-\frac{1}{4}}}$$

$$R_{(n+1)}(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$$

$$R_3(x) = \frac{f'''(\xi)}{3!}(x-x_0)^3 \Rightarrow R_3(x) = \frac{16}{6}\frac{1}{(2\xi-1)^3}(x-1)^3 = \frac{8}{3}\frac{1}{(2\xi-1)^3}(x-1)^3$$

$\xi \in \left(1, \frac{5}{4}\right)$

$$\left|f\left(\frac{5}{4}\right) - T\left(\frac{5}{4}\right)\right| \leq R_3\left(\frac{5}{4}\right)$$

$$R_3\left(\frac{5}{4}\right) = \frac{8}{3}\frac{1}{(2\cdot 1-1)^3}\left(\frac{5}{4}-1\right)^3 = \frac{8}{3}\cdot\frac{1}{64} = \frac{1}{3\cdot 8} = \frac{1}{24}$$

$$\left|R_3\left(\frac{5}{4}\right)\right| \leq \frac{1}{24}$$

2\. $f(x) = x^2 e^{x-2}$ — rovnice tečny ke grafu v bodě $x_0=2$

a) $y - y_0 = f'(x_0)(x - x_0)$

$y_0 = f(x_0) = 4 \cdot 1 = 4$

$f'(x) = 2x \cdot e^{x-2} + x^2 \cdot 1 e^{x-2}$

$f'(2) = 4 + 4 = 8$

$$\underline{y = 4 + 8(x-2)}$$

b) $f(x,y) = x + 4\sqrt{x} - 2y$ $\quad x \in (0,9\rangle$

$y = x - 1$

$f(x) = x + 4\sqrt{x} - 2(x-1)$

$f'(x) = 1 + 4 \cdot \frac{1}{2} x^{-\frac{1}{2}} - 2 = -1 + 2\frac{1}{\sqrt{x}}$

$-1 + \frac{2}{\sqrt{x}} = 0$

$\frac{2}{\sqrt{x}} = 1$

$\sqrt{x} = 2$

$x = 4$

$f(4,3) = 4 + 4\sqrt{4} - 2 \cdot 3 = 6$ max

$f(0,-1) = 2$ min

– zadaná fce je ~~omezená~~ spojitá na dané úsečce, což je množina omezená ~~uzavřená~~ a uzavřená v E_2

3\. a) $\ddot{x} - 5\dot{x} + 6x = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$D = 25 - 4 \cdot 6 = 1$

$\lambda_{1,2} = \frac{5 \pm 1}{2} = \begin{cases} 3 \\ 2 \end{cases}$

$\lambda_1 = 3$

$\lambda_2 = 2$

– fundamentální systém

$\{ \varphi_1(t) = e^{3t} ; \varphi_2(t) = e^{2t} \}$

$x(t) = C_1 e^{3t} + C_2 e^{2t}$; $t \in (-\infty, +\infty)$

$\dot{x}(t) = 3C_1 e^{3t} + 2C_2 e^{2t}$

– maximální řešení pro $x(0) = 2$; $\dot{x}(0) = 1$

$2 = C_1 + C_2 \quad /(-3)$

$1 = 3C_1 + 2C_2$

$-5 = -C_2 \qquad C_1 = -3$

$C_2 = 5$

$x(t) = -3e^{3t} + 5e^{2t}$

$t \in (-\infty, +\infty)$

b) $\ddot{x} + 4x = 0$ — tvar pravé strany — $2t\,e^{3t}$

— $3\sin 2t$

$\lambda^2 + 4 = 0$

$\lambda^2 = -4$

$(\lambda - 2i)(\lambda + 2i) = 0$

$\lambda_1 = 2i$

$\lambda_2 = -2i$

$\lambda_{1,2} = \underset{0}{\underset{\parallel}{\alpha}} \pm i\underset{2}{\underset{\parallel}{\beta}}$

$x(t) = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$

$x(t) = C_1 \cos 2t + C_2 \sin 2t$

— fundamentální systém

$\{\varphi_1(t) = \cos 2t \,;\ \varphi_2(t) = \sin 2t\}$

— : $\ddot{x} + 4x = 2t \cdot e^{3t}$

$x_p = (At + B)e^{3t}$

$t \in (-\infty; +\infty)$

— : $\ddot{x} + 4x = 3\sin 2t$

$x_p = (A \cdot \cos 2t + B \sin 2t)\,t$

$t \in (-\infty, +\infty)$

2018/2019

1. $f(x)=\sqrt{2x+4}=(2x+4)^{\frac{1}{2}}$

- rovnice tečny $y-y_0=f'(x)(x-x_0)$

$x_0=0$

$\boxed{y=2+\frac{1}{2}x=\frac{x}{2}+2}$

a) $2x+4\geq 0$

$x\geq -2$

$\mathcal{D}(f)=\langle -2,+\infty)$

$f'(x)=\frac{1}{2}\cdot(2x+4)^{-\frac{1}{2}}\cdot 2=\frac{1}{\sqrt{2x+4}}$

$f'(0)=\frac{1}{2}$

$f''(x)=\frac{-1\cdot\frac{1}{2}(2x+4)^{-\frac{1}{2}}\cdot 2}{2x+4}=\frac{-1}{(2x+4)\sqrt{2x+4}}=\frac{-1}{(2x+4)^{\frac{3}{2}}}$

$f''(0)=\frac{-1}{4\cdot 2}=-\frac{1}{8}$

$f'''(x)=\frac{1\cdot\frac{3}{2}(2x+4)^{\frac{1}{2}}\cdot 2}{(2x+4)^3}=\frac{3(2x+4)^{\frac{1}{2}}}{(2x+4)^3}=\frac{3}{(2x+4)^{\frac{5}{2}}}$

$f'''(0)=\frac{3\cdot 2}{4\cdot4\cdot4}=\frac{3}{32}$

$f(0)=2$

b) $T_2(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2$ $\quad x_0=0$

$T_2(x)=2+\frac{1}{2}x-\frac{1}{2}\cdot\frac{1}{8}x^2=2+\frac{1}{2}x-\frac{1}{16}x^2$

$f(\tfrac{1}{2})\doteq T_2(\frac{1}{2})=2+\frac{1}{2}\cdot\frac{1}{2}-\frac{1}{16}\cdot\frac{1}{4}=2+\frac{1}{4}-\frac{1}{64}=\frac{128+16-1}{64}=\frac{143}{64}$

c) $R_3(x)=\frac{3\cdot(2\xi+4)^{-\frac{5}{2}}}{3!}(x)^3=\frac{1}{2(2\xi+4)^{\frac{5}{2}}}x^3$

ξ leží mezi 0 a $\frac{1}{2}$

$x_0=0$

$\Downarrow$

za ξ se dosadí 0

za x se dosadí $\frac{1}{2}$

$R_3(\frac{1}{2})=\frac{1}{2}\frac{1}{(2\cdot 0+4)^{\frac{5}{2}}}\cdot\frac{1}{8}=\frac{1}{2}\cdot\frac{1}{\sqrt{4^5}}\cdot\frac{1}{8}=\frac{1}{16}\cdot\frac{1}{2^5}=\frac{1}{512}$

$\sqrt{4^5}=\sqrt{4\cdot4\cdot4\cdot4\cdot4}=$
$=2^5$

$\underline{|R_3(\frac{1}{2})|\leq\frac{1}{512}}$

② a) vlastní čísla

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

$\det(A - \lambda E) = 0$

$$\det \begin{vmatrix} 2-\lambda & -1 & 3 \\ 0 & 1-\lambda & 2 \\ 0 & 2 & 1-\lambda \end{vmatrix} = 0 = (2-\lambda)(1-\lambda)(1-\lambda) + 0 + 0 - 0 - 4(2-\lambda) - 0 = 0$$

$$(2-\lambda)\left[(1-\lambda)(1-\lambda) - 4\right] = 0$$

$1 - 2\lambda + \lambda^2 - 4 = 0$ | $D = 4 - 4(-3)$

$\lambda^2 - 2\lambda - 3 = 0$ | $D = 16$

$\lambda_{2,3} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \end{cases}$

$\underline{\lambda_1 = 2}$

$\underline{\lambda_2 = 3}$

$\underline{\lambda_3 = -1}$

– spektrální poměr $\varrho(A)$ – největší z vlastních čísel

$\underline{\varrho(A) = 3 = \varrho(\lambda_2)}$

b) $\lambda_3 = -1$.

$$\begin{pmatrix} 3 & -1 & 3 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$

$u_3 = v$

$2u_2 + 2u_3 = 0$

$u_2 = -v$

$3u_1 - u_2 + 3u_3 = 0$

$3u_1 + v + 3v = 0$

$u_1 = -\frac{4}{3} v$

$$\underline{u} = \begin{pmatrix} -4 \\ -3 \\ 3 \end{pmatrix} v$$

$\lambda_2 = 3$

$$\begin{pmatrix} -1 & -1 & 3 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{pmatrix}$$

$u_3 = v$

$-2u_2 + 2u_3 = 0$

$u_2 = v$

$-u_1 - u_2 + u_3 \cdot 3 = 0$

$-u_1 - v + 3v = 0$

$u_1 = 2v$

$$\underline{u} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} v$$

③ $\dot{x} = \underbrace{-2y(x+1)}_{P(x,y)}$ $\dot{y} = \underbrace{x+y^2}_{Q(x,y)}$ – autonomní soustava

a) fázová trajektorie soustavy

$$J = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix} = \begin{pmatrix} -2y & -2(x+1) \\ 1 & 2y \end{pmatrix}$$

Jacobiho matice je spojitá v E_2

b) body rovnováhy

– tam kde $P(x,y)=0$ a $Q(x,y)=0$

$-2y(x+1)=0$ $\quad$ $x+y^2=0$

$\to y=0$ $\quad$ $\to$ pro $y=0 \to x=0$

$\to x=-1$ $\quad$ – pro $x=-1 \to y^2-1=0$

$(y-1)(y+1)=0$

1. bod $[0,0]$
2. bod $[-1,-1]$
3. bod $[-1,1]$

c) rovnice fáz. trajektorií

– pro bod $M=[-1,2]$

autonomní rovnice:

$+Q(x,y)\,dx + P(x,y)\,dy = 0$

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)} = \frac{x+y^2}{-2y(x+1)}$$

$-(x+y^2)\,dx - 2y(x+1)\,dy = 0$

$(x+y^2)\,dx + 2y(x+1)\,dy = 0$

→ jde o rovnici exaktní a tak ji řešíme, protože:

$\frac{\partial}{\partial x} 2y(x+1) = \frac{\partial}{\partial y}(x+y^2)$

$$\frac{\partial h}{\partial x} = Q(x,y) = x+y^2$$

$$\frac{\partial h}{\partial y} = P(x,y) = +2y(x+1)$$

$h(x,y) = \int Q(x,y)\,dx = \int (x+y^2)\,dx = x^2 \cdot \frac{1}{2} + y^2 x + K(y)$

$$\frac{\partial h(x,y)}{\partial y} = 2yx + \frac{\partial K(y)}{\partial y}$$

$$2y(x+1) = 2yx + \frac{dK(y)}{dy}$$

$$2yx + 2y = 2yx + \frac{dK(y)}{dy}$$

$$2y\,dy = dK(y)$$

$$K(y) = y^2 + C$$

$$\rightarrow h(x,y) = \frac{x^2}{2} + y^2 x + K(y)$$

$$h(x,y) = \frac{x^2}{2} + y^2 x + y^2 + C = 0$$

- vyjádření pro $M = [-1, 2]$

$$0 = \frac{(-1)^2}{2} + 2^2 \cdot (-1) + 2^2 + C = 0$$

$$\frac{1}{2} - 4 + 4 + C = 0$$

$$C = -\frac{1}{2}$$

- rovnice fáz. trajektorií

$$\frac{x^2}{2} + y^2 x + y^2 + C = 0$$

- rovnice fáz. trajektorie procházející bodem M

$$\frac{x^2}{2} + y^2 x + y^2 - \frac{1}{2} = 0$$

2017/2018

① $f(x) = \frac{x^2+4}{x} + 2$ $\qquad f'(x) = \frac{2x^2 - x^2 - 4}{x^2} = \frac{x^2-4}{x^2}$

a) $x \neq 0 \quad \mathcal{D}(f) = (-\infty, 0) \cup (0, +\infty)$ $\qquad f''(x) = \frac{2x^3 - 2x(x^2-4)}{x^4} = \frac{-4}{x^4}$

$x_0 = 4$

$f(4) = \frac{4 \cdot 4 + 4}{4} + 2 = 7$

$f'(4) = \frac{44-4}{4 \cdot 4} = \frac{12}{16} = \frac{3}{4}$

$y - f(x_0) = f'(x_0)(x - x_0)$ $\qquad f(x) = y = 7 + \frac{3}{4}(x-4)$

$\underline{y = 7 + \frac{3}{4}(x-4)}$ $\qquad f\left(\frac{7}{2}\right) = 7 + \frac{3}{4}\left(\frac{7}{2} - 4\right) = 7 + \frac{3}{4}\frac{(-1)}{2} = \frac{56-3}{8} = \underline{\frac{53}{8}}$

b) $I = \langle -3, -1 \rangle$

$\frac{x^2-4}{x^2} = 0$ $\qquad x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$\downarrow \qquad \downarrow$

$x = 0 \leftarrow$ mimo interval $\rightarrow x = 2 \qquad x = -2$

$f(-3) = \frac{(-3)^2+4}{-3} + 2 = -\frac{13}{3} + \frac{6}{3} = \frac{-7}{3} \doteq -2{,}3$

$f(-2) = \frac{(-2)^2+4}{-2} + 2 = -2$

$f(-1) = \frac{1+4}{-1} + 2 = -5 + 2 = -3$

maximum $f(-2) = -2$

minimum $f(-1) = -3$

② $A = \begin{pmatrix} 1 & 2 & -5 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$ $\qquad \det(A - \lambda E) = 0 = \begin{vmatrix} 1-\lambda & 2 & -5 \\ 0 & 2-\lambda & 0 \\ 1 & 2 & 3-\lambda \end{vmatrix}$

a) vlastní čísla

$0 = (1-\lambda)(2-\lambda)(3-\lambda) + 0 + 0 + 5(2-\lambda) - 0 - 0$

$0 = (2-\lambda)[(1-\lambda)(3-\lambda) + 5]$

$\underline{\lambda_1 = 2}$ $\qquad (1-\lambda)(3-\lambda) + 5 = 0$

$3 - 4\lambda + \lambda^2 + 5 = 0$

$\lambda^2 - 4\lambda + 8 = 0$

$D = 16 - 4 \cdot 8 < 0$

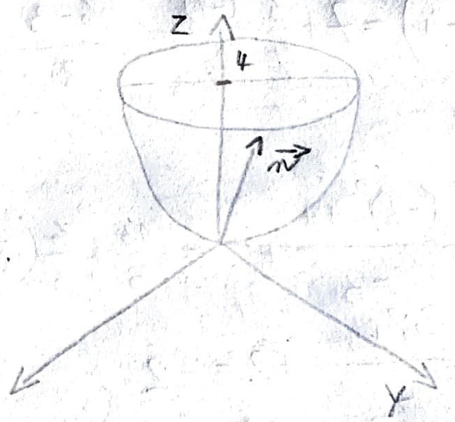
$$\begin{pmatrix} -1 & 2 & -5 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix} \qquad v_3 = r \qquad \begin{array}{r} v_1 + 2v_2 + r = 0 \\ -v_1 + 2v_2 - 5r = 0 \\ \hline 4v_2 - 4r = 0 \end{array}$$

$$v_1 + 2r + r = 0 \qquad\qquad v_2 = r$$

$$v_1 = -3r$$

$$\underline{v} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} r$$

③ a) $Q = \{[x,y,z] \in E_3;\ z = x^2+y^2;\ z \le 4\}$



$P(u,v) = [u, v, u^2+v^2]$

$$\left[\begin{array}{l} x = u \\ y = v \\ z = x^2+y^2 = u^2+v^2 \end{array}\right.$$

$P_u = [1, 0, 2u)$

$P_v = (0, 1, 2v)$

$$\vec{n} = P_u \times P_v = \begin{vmatrix} i & j & k \\ 1 & 0 & 2u \\ 0 & 1 & 2v \end{vmatrix} = 0\cdot i + k + 0j - 0k - 2u\cdot i - 2vj = -2ui - 2vj + k$$

$z = x^2+y^2;\ [x,y] \in B$

$B = \{[x,y] \in E_2;\ x^2+y^2 \le 4\}$

$P_u \times P_v = (-2u, -2v, 1)$

-tok

$$\iint_Q f(x,y,z)\,dp = \iint_B f(P(u,v))\cdot(P_u \times P_v)\,du\,dv =$$

$$= \iint_B (\cancel{x}^{u}, \cancel{y}^{v}, 0)(-2u, -2v, 1)\,du\,dv = \iint_B (-2u^2 - 2v^2)\,du\,dv =$$

$$\left|\begin{array}{l} u = r\cdot\sin\varphi = v \\ v = r\cdot\cos\varphi = u \\ du\,dv = r\,dr\,d\varphi \end{array} \;\vdots\; \begin{array}{l} \overset{\text{poloměr}}{0 \le r \le 2} \\ 0 \le \varphi \le 2\pi \\ \end{array}\right| = \int_0^{2\pi}\int_0^{2} (-2)\underbrace{(r^2\cos^2\varphi + r^2\sin^2\varphi)}_{=1}\, r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^{2} (-2)\, r^3\, dr\, d\varphi = -2 \cdot 2\pi \left[\frac{r^4}{4}\right]_0^2 = -4\pi\, \frac{16}{4} = \underline{-16\pi}$$

b) $\vec{f} = (xy^2,\, x+z,\, x^2 \cdot z) \; \rightarrow \; \vec{f} = (U, V, W)$

orientace:
vně $\oplus$
$\boxed{\text{dovnitř } \ominus}$

- srovnání: $\iint_Q \vec{f}\, d\vec{p} = + \iiint_{\text{int } Q} \operatorname{div} \vec{f}\; dx\,dy\,dz$

$M = \left\{ [x,y,z] \in E_3;\ \begin{matrix} x^2+y^2 \le 4 \\ 0 \le z \le 3 \end{matrix} \right\}$

$$\operatorname{div} \vec{f} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = y^2 + 0 + x^2$$

$$\boxed{-}\iint_Q \vec{f}\, d\vec{p} = \boxed{-}\iiint_{\substack{x^2+y^2 \le 4 \\ 0 \le z \le 3}} x^2 + y^2\, dx\,dy\,dz = \left| \begin{array}{l|l} x = r \cdot \cos\varphi & 0 \le r \le 2 \\ y = r \cdot \sin\varphi & 0 \le \varphi \le 2\pi \\ z = z & 0 \le z \le 3 \\ \multicolumn{2}{c}{dx\,dy\,dz = r\,dr\,d\varphi\,dz} \end{array} \right| =$$

$$= -\int_0^3\int_0^{2\pi}\int_0^2 (x^2+y^2)\, r\,dr\,d\varphi\,dz = -\int_0^3\int_0^{2\pi}\int_0^2 r^2 \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1}\, r\,dr\,d\varphi\,dz = -2\pi \cdot 3 \cdot \left[\frac{r^4}{4}\right]_0^2 =$$

$$= -6\pi \cdot \frac{4 \cdot 4}{4} = \underline{-24\pi}$$

2016/2017

1. $f(x) = (x-6)\sqrt{x}$

$$f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} \cdot (x-6) + 1 \cdot \sqrt{x}$$

a) $x \geq 0$

$$f'(4) = \sqrt{4} + \frac{1}{2} \frac{1}{\sqrt{4}} \cdot (4-6) = 2 + \frac{1}{4}(-2) = \frac{3}{2}$$

$D(f) = \langle 0, +\infty)$

$x_0 = 4$

$$y - y_0 = f'(x_0)(x - x_0)$$

$y_0 = f(x_0) = (4-6)\sqrt{4} = -4$

$$y = -4 + \frac{3}{2}(x-4)$$

$$y = \frac{3}{2}x - 6 - 4 = \frac{3}{2}x - 10$$

$$y\left(\frac{9}{2}\right) = \frac{3}{2}\,\frac{9}{2} - 10 = \frac{27-40}{4} = -\frac{13}{4} \doteq f\left(\frac{9}{2}\right)$$

b) $I \in \langle 0, 4 \rangle$

$$f'(x) = \frac{x-6}{2\sqrt{x}} + \sqrt{x} = 0$$

$$\frac{\sqrt{x}}{\sqrt{x}} \cdot \frac{(x-6) + 2x}{2\sqrt{x}} = 0$$

$$\frac{\sqrt{x}[3x-6]}{2x} = 0$$

$x \neq 0$

$3x - 6 = 0$

$x = 2$

$f(2) = (2-6)\sqrt{2} = -4\sqrt{2}$

$f(0) = 0$

$f(4) = -4$

- max $f(0) = 0$
- min $f(2) = -4\sqrt{2}$

- zadaná fce je na daném intervalu spojitá, interval je omezený a uzavřený

② $\ddot{x} + 3\dot{x} + 2x = 5e^{-2t}$

a) $\ddot{x} + 3\dot{x} + 2x = 0$

$\lambda^2 + 3\lambda + 2 = 0$

$D = 9 - 4\cdot 2 = 1$

$\lambda_{1,2} = \frac{-3 \pm 1}{2} = \begin{cases} -2 \\ -1 \end{cases}$

$x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$

– fundamentální systém

$\underline{\{ \varphi_1(t) = e^{-2t} ;\ \varphi_2(t) = e^{-t} \}}$

$\underline{x(t) = c_1 e^{-2t} + c_2 e^{-t} \quad ; t \in (-\infty; +\infty)}$

b) $\underline{x_p = A e^{-2t} \cdot t}$

$\dot{x}_p = A \cdot (-2) e^{-2t} \cdot t + A e^{-2t}$

$\ddot{x}_p = -2A(-2) e^{-2t} \cdot t - 2A e^{-2t} \cdot 1 - 2A e^{-2t} = 4A e^{-2t} \cdot t - 4A e^{-2t}$

$+ 4tA e^{-2t} - 4A e^{-2t} - 6A e^{-2t} \cdot t + 3A e^{-2t} + 2tA e^{-2t} = 5e^{-2t}$

$-4A e^{-2t} + 3A e^{-2t} = 5e^{-2t}$

$A = -5$

c) $\underline{x_p = -5e^{-2t} \cdot t}$

$\underline{x(t) = c_1 e^{-2t} + c_2 e^{-t} - 5e^{-2t} \cdot t ;\ t \in (-\infty, +\infty)}$

$x(0) = 5$

$\dot{x}(0) = -15$

$\dot{x}(t) = -2c_1 e^{-2t} - c_2 e^{-t} + 10 e^{-2t} \cdot t - 5e^{-2t}$

$5 = c_1 + c_2 - 5 \cdot 0$

$-15 = -2c_1 - c_2 - 5$

$5 = c_1 + c_2 \quad \Rightarrow \quad 5 = 5 + c_2$

$-10 = -2c_1 - c_2 \qquad c_2 = 0$

$-5 = -c_1$

$c_1 = 5$

$\underline{x(t) = 5e^{-2t} - 5e^{-2t} \cdot t \ ;\ t \in (-\infty, +\infty)}$

! λ_1 se shoduje s exponentem u e !!!

x_p pak musí být $x_p = A \cdot t \cdot e^{-2t}$!!!

3.
$$\dot{x} = x^2 + y^3 \rightarrow P(x,y)$$
$$\dot{y} = -2x(y+1) \rightarrow Q(x,y)$$

a) $$J = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix} = \begin{pmatrix} 2x & 3y^2 \\ -2(y+1) & -2x \end{pmatrix}$$

Jacobiho matice je spojitá v E_2

b) $P(x,y) = 0 = x^2 + y^3$ — pro $x=0$ je $y=0$

$Q(x,y) = 0 = -2x(y+1)$ — pro $y=-1$ je $x^2-1=0$

$x=0 \quad y=-1$ $\qquad$ $x=-1;\ x=1$

1. bod $[0,0]$
2. bod $[-1,-1]$
3. bod $[+1,-1]$

c) $$\frac{dy}{dx} = \frac{-2x(y+1)}{x^2+y^3} \qquad (x^2+y^3)\,dy = -2x(y+1)\,dx \qquad M = [-2,3]$$

$$(x^2+y^3)\,dy + 2x(y+1)\,dx = 0$$

$$\frac{\partial h}{\partial x} = Q(x,y) = 2x(y+1)$$

$$\frac{\partial h}{\partial y} = P(x,y) = x^2 + y^3$$

$$h(x,y) = \int Q(x,y) = \int 2x(y+1)\,dx = x^2(y+1) + K(y)$$

$$\frac{\partial h(x,y)}{\partial y} = x^2 + \frac{\partial K(y)}{\partial y}$$

$$x^2 + y^3 = x^2 + \frac{\partial K(y)}{\partial y}$$

$$K(y) = \frac{y^4}{4} + C$$

$$h(x,y) = x^2(y+1) + \frac{y^4}{4} + C = 0$$

$$4(4) + \frac{3^4}{4} + C = 0 \Rightarrow C = -\frac{145}{4}$$

$$x^2(y+1) + \frac{y^4}{4} - \frac{145}{4} = 0$$

2015/2016

1. $f(x)=2\sqrt{x-1}-x$ $\qquad f(5)=2\sqrt{4}-5=-1$

a) $x-1\geq 0$ $\quad \mathcal{D}(f)=\langle 1,+\infty)$ $\qquad f'(x)=2\cdot\frac{1}{2}(x-1)^{-\frac{1}{2}}-1=\frac{1}{\sqrt{x-1}}-1$

$x\geq 1$ $\qquad f'(5)=-\frac{1}{2}$

$x_0=5$

$$y-f(x_0)=f'(x_0)(x-x_0)$$

$$\underline{y}=-1+\left(-\frac{1}{2}\right)(x-5)=-\frac{x}{2}+\frac{5}{2}-1=\underline{\frac{3}{2}-\frac{x}{2}}$$

$$\underline{f\left(\frac{11}{2}\right)}\doteq y\left(\frac{11}{2}\right)=\frac{3}{2}-\frac{1}{2}\cdot\frac{11}{2}=\frac{6-11}{4}=\underline{-\frac{5}{4}}$$

b) $I=\langle 1,6\rangle$

$$f'(x)=\frac{1-\sqrt{x-1}}{\sqrt{x-1}}=0 \rightarrow 1-\sqrt{x-1}=0$$

$1=\sqrt{x-1}$

$1=x+1(-1)$

$x=2$

$$\frac{1-\sqrt{x-1}}{\sqrt{x-1}}\cdot\frac{\sqrt{x-1}}{\sqrt{x-1}}=0$$

$$\frac{\sqrt{x-1}-x+1}{x-1}=0$$

$f(1)=-1$

$f(6)=2\cdot\sqrt{5}-6$

$f(2)=2-2=0$

max $f(2)=0$

$\underline{\text{min } f(6)=2\sqrt{5}-6}$

-funkce je spojitá na daném intervalu, který je omezený a uzavřený

2. a) $A=\begin{pmatrix}3&5&2\\-1&5&3\\0&0&2\end{pmatrix}$ $\qquad \det(A-\lambda E)=\begin{vmatrix}3-\lambda&5&2\\-1&5-\lambda&3\\0&0&2-\lambda\end{vmatrix}=0$

$$(3-\lambda)(5-\lambda)(2-\lambda)+0+0-0-0+5(2-\lambda)=0$$

$\underline{\lambda_1=2}$ $\qquad (3-\lambda)(5-\lambda)+5=0$

$\underline{\lambda_2=4+2i}$ $\qquad 15-3\lambda-5\lambda+\lambda^2+5=0$

$\underline{\lambda_3=4-2i}$ $\qquad \lambda^2-8\lambda+20=0$

$D=64-4\cdot 20=-16$

$\lambda_{2,3}=\frac{8\pm 4i}{2}=$

$\begin{pmatrix}1&5&2\\-1&3&3\\0&0&0\end{pmatrix}$ $\quad u_3=v$

$-u_1+3u_2+3v=0$

$\underline{u_1+5u_2+2v=0}$

$u_1-\frac{25}{8}v+2v=0 \qquad 8u_2+5v=0$

$u_1=\frac{25-16}{8}v=\frac{9}{8}v \qquad u_2=-\frac{5}{8}v$

$\underline{u=(9,-5,8)^T v}$

③ $\dot{x} = 2y(x+2) \rightarrow P(x,y)$

$\dot{y} = x^2 - y^2 \rightarrow Q(x,y)$

a) $J = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix} = \begin{pmatrix} 2y & 2(x+2) \\ 2x & -2y \end{pmatrix}$ je spojitá v E_2

b) $P(x,y) = 0 = 2y(x+2) \rightarrow y = 0$ a $x = -2$

$Q(x,y) = 0 = x^2 - y^2$

- pro $y = 0$ je $x = 0$
- pro $x = -2$ je $y^2 - 4 = 0$

$(y-2)(y+2) = 0$

1. bod $[0,0]$
2. bod $[-2,-2]$
3. bod $[-2,2]$

c) $M = [3,1]$

$\frac{dy}{dx} = \frac{x^2 - y^2}{2y(x+2)}$

$2y(x+2)\,dy = (x^2 - y^2)\,dx$

$2y(x+2)\,dy + (y^2 - x^2)\,dx = 0$

$\frac{\partial h}{\partial y} = P(x,y) = 2y(x+2)$

$\frac{\partial h}{\partial x} = Q(x,y) = y^2 - x^2$

$h(x,y) = \int Q(x,y)\,dx = \int (y^2 - x^2)\,dx =$

$= y^2 x - \frac{x^3}{3} + K(y)$

$\frac{\partial h(x,y)}{\partial y} = 2yx + \frac{dK(y)}{dy}$

$2y(x+2) = 2yx + \frac{dK(y)}{dy}$

$4y\,dy = dK(y)$

$K(y) = 2y^2 + C$

$h(x,y) = y^2 x - \frac{x^3}{3} + 2y^2 = 0$

$y^2 x - \frac{x^3}{3} + 2y^2 + c = 0$

$1 \cdot 3 - 9 + 2 \cdot 1 + c = 0$

$c = 4$

$y^2 x - \frac{x^3}{3} + 2y^2 + c = 0$

$y^2 x - \frac{x^3}{3} + 2y^2 + 4 = 0$

2014/2015

1. $f(x) = e^{3x-6} - x$ $\qquad f(2) = e^1 - 2 = -1$

a) $x_0 = 2$ $\qquad f'(x) = e^{3x-6} \cdot (3) - 1 = 3e^{3x-6} - 1 \quad \Big| \quad f'(2) = 3 \cdot 1 - 1 = 2$

$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$y = -1 + 2(x-2) = -1 + 2x - 4 = 2x - 5$$

b) $T_2(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 \qquad x_0 = 2$

$f''(x) = 3 \cdot e^{3x-6} \cdot 3 = 9e^{3x-6}$

$f''(x_0) = 9$

$T_2(x) = -1 + 2(x-2) + \frac{9}{2}(x-2)^2$

$T_2\left(\frac{5}{3}\right) = -1 + 2\left(\frac{5}{3} - 2\right) + \frac{9}{2}\left(\frac{5}{3} - 2\right)^2 = -1 + 2\left(-\frac{1}{3}\right) + \frac{9}{2}\left(-\frac{1}{3}\right)^2 = -\frac{5}{3} + \frac{1}{2} = \frac{-10+3}{6} = -\frac{7}{6}$

$f\left(\frac{5}{3}\right) \doteq -\frac{7}{6}$

c)

$f'''(x) = 9 \cdot e^{3x-6} \cdot 3 = 27e^{3x-6}$

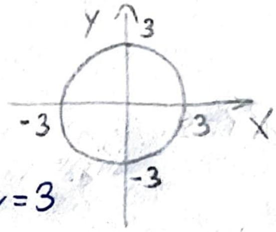
$R_3(x) = \frac{f'''(\xi)}{3!}(x - x_0)^3 = \frac{9e^{3\xi-6}}{2}(x-2)^3 \quad \xi$ je mezi 2 a $\frac{5}{3}$

$R_3\left(\frac{5}{3}\right) = \frac{9e^{6-6}}{2}\left(\frac{5}{3} - 2\right)^3 = \frac{9}{2}\left(\frac{5-6}{3}\right)^3 = \frac{9}{2}\left(-\frac{1}{3}\right)^3 = -\frac{9}{2} \cdot \frac{1}{27} = -\frac{1}{6}$

$\left|R_3\left(\frac{5}{3}\right)\right| \leq +\frac{1}{6}$

(2.) křivka $C = \{[x,y] \in E_2 ; x^2+y^2=9\}$

↳ rovnice kružnice o $r=3$



a) $\oint_C \vec{f}\, d\vec{s} = \iint_{int\, C} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx dy$ $\qquad \frac{\partial Q}{\partial x} = 1 \quad \frac{\partial P}{\partial y} = -2$

$\boxed{f = (x-2y, x)}$

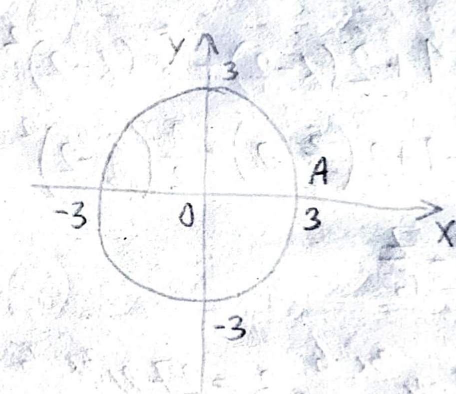
↳ $P(x,y) = x - 2y$

$Q(x,y) = x$

$\oint_C \vec{f}\, d\vec{s} = \iint_{int\, C} (1-(-2))\, dx dy = \left| \begin{array}{l|l} x = r.\cos\varphi & 0 \le r \le 3 \\ y = r.\sin\varphi & 0 \le \varphi \le 2\pi \\ dx dy = r dr d\varphi & \end{array} \right|$

$= \int_0^{2\pi}\int_0^3 3r dr d\varphi = 3 \cdot 2\pi \cdot \left[\frac{r^2}{2}\right]_0^3 = 6\pi \cdot \frac{9}{2} = 27\pi$

b) $\oint_C \vec{f}\, d\vec{s} = \int_{C_0} \vec{f}\, d\vec{s}$



- křivka $x^2+y^2=9$
- počáteční bod $A = [3,0]$
- parametrizace $x = r.\cos t = 3\cos t$
$y = r.\sin t = 3 \sin t$ $\quad t \in [0, 2\pi)$

$P_1(t) = [3\cdot\cos t, 3\sin t] \rightarrow P_1(0) = [3\cdot 1 ; 3\cdot 0] = A$ - orientace souhlasná

$P^{\cdot}(t) = [-3\sin t ; 3\cos t]$

$f(x-2y, x) = f(3\cos t - 6\sin t ; 3\cdot\cos t)$

$\int_C \vec{f}\, d\vec{s} = \int_0^{2\pi} (3\cos t - 6\sin t ; 3\cos t)(-3\sin t ; 3\cos t)\, dt =$

$= \int_0^{2\pi} (-9\cos t \sin t + 18\sin^2 t + 9\cos^2 t)\, dt$

$$= \int_0^{2\pi} -9\cos t \sin t\, dt + \int_0^{2\pi} 18\sin^2 t\, dt + \int_0^{2\pi} 9\cdot\cos^2 t\, dt =$$

$$-9\int_0^{2\pi} \cos t \sin t\, dt = \left| \begin{array}{l} x = \sin t \\ dx = \cos t\, dt \\ \frac{dx}{\cos t} = dt \end{array} \right| = -9\int_0^{2\pi} \cos t \cdot x \cdot \frac{dx}{\cos t} = -9\left[\frac{x^2}{2}\right]_0^{2\pi} = -9\left[\frac{\sin^2 t}{2}\right]_0^{2\pi}$$

$$= -9\cdot 0 = \underline{0}$$

$$\frac{1-\cos 2t}{2}$$

$$18\int_0^{2\pi} \sin^2 t\, dt = 18\int_0^{2\pi} \frac{1-\cos 2t}{2}\, dt = \left| \begin{array}{l} x = 2t \\ dx = 2\, dt \\ \frac{1}{2} dx = dt \end{array} \right| = 18\int_0^{2\pi} \frac{1-\cos x}{2}\cdot\frac{1}{2}\, dx =$$

$$= 18\cdot\frac{1}{2}\int_0^{2\pi}\left(\frac{1}{2} - \frac{\cos x}{2}\right)dx = 18\cdot\frac{1}{2}\cdot\frac{1}{2}\int_0^{2\pi}(1-\cos x)\, dx = \frac{9}{2}\left[x - \sin x\right]_0^{2\pi} =$$

$$= \frac{9}{2}\left[2t - \sin 2t\right]_0^{2\pi} = \frac{9}{2}\cdot 4\pi - 0 = \underline{18\pi}$$

$$9\int_0^{2\pi} \cos^2 t\, dt = \left| \begin{array}{l} 2t = x \\ dx = 2\, dt \\ \frac{1}{2} dx = dt \end{array} \right| = \frac{9}{2}\int_0^{2\pi}(1+\cos x)\frac{1}{2}\, dx = \frac{9}{4}\left[x + \sin x\right]_0^{2\pi} =$$

$$\frac{1+\cos 2t}{2}$$

$$= \frac{9}{4}\left[2t + \sin 2t\right]_0^{2\pi} = \frac{9}{4}\cdot 4\pi + 0 = \underline{9\pi}$$

$$= 0 + 18\pi + 9\pi = \underline{27\pi}$$

③ a) $\ddot{x}+2\dot{x}-3x=0$

$\lambda^2+2\lambda-3=0$

$D=4-4(-3)=16$

$\lambda_{1,2}=\frac{-2\pm4}{2}=\begin{cases}1\\-3\end{cases}$

$\lambda_1=1$; $\lambda_2=-3$

– fundamentální systém

$\{\varphi_1(t)=e^{t};\ \varphi_2(t)=e^{-3t}\}$

– obecné řešení

$x(t)=c_1e^{t}+c_2e^{-3t}$ $t\in(-\infty,+\infty)$

$\dot{x}(t)=c_1e^{t}-3c_2e^{-3t}$

$x(0)=6$

$\dot{x}(0)=0$

$0=c_1-3c_2 \quad /\cdot(-1)$

$6=c_1+c_2$

$6=4c_2$

$c_2=\frac{3}{2}$

$c_1=6-\frac{3}{2}=\frac{12-3}{2}=\frac{9}{2}$

– maximální řešení

$x(t)=\frac{9}{2}e^{t}+\frac{3}{2}e^{-3t}$ $t\in(-\infty,+\infty)$

b) $\ddot{x}+2\dot{x}-3x=15e^{-3t}$

$x_p=A\cdot e^{-3t}\cdot t$

$\dot{x}_p=-3Ae^{-3t}\cdot t+Ae^{-3t}\cdot 1$

$\ddot{x}_p=9Ae^{-3t}\cdot t-3Ae^{-3t}-3Ae^{-3t}=$

$=9tAe^{-3t}-6Ae^{-3t}$

$9tAe^{-3t}-6Ae^{-3t}-6Ae^{-3t}\cdot t+2Ae^{-3t}-$

$-3Ae^{-3t}t=15e^{-3t}$

$-6Ae^{-3t}+2Ae^{-3t}=15e^{-3t}$

$-6A+2A=15$

$A=-\frac{15}{4}$

$x_p(t)=-\frac{15}{4}te^{-3t}$

$x(t)=c_1e^{t}+c_2e^{-3t}-\frac{15}{4}\cdot te^{-3t}$, $t\in(-\infty,+\infty)$

! λ_2 se shoduje s exponentem u e ! x_p pak musí být $x_p=A\cdot t\cdot e^{-3t}$!!!

2013/2014

(1.) $f(x) = \ln(x+1) - \frac{1}{2}x^2$

$f'(x) = \frac{1}{x+1} \cdot 1 - \frac{1}{2} \cdot 2x = \frac{1}{x+1} - x$

a) $x_0 = 0$

$f'(0) = 1$

$y - f(x_0) = f'(x_0)(x - x_0)$

$f(0) = 0$

$y = 0 + 1(x-0) = x$

$f''(x) = \frac{-1(1}{(x+1)^2} - 1 = \frac{-1}{(x+1)^2} - 1$

$f''(0) = -2$

b) $x_0 = 0$

$T_2(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2$

$T_2(x) = 0 + (x - 0) + \frac{-2}{2}x^2 = x - x^2$

$f(\frac{1}{3}) \doteq T_2(\frac{1}{3}) = \frac{1}{3} - \frac{1}{9} = \frac{9-3}{27} = \frac{6}{27} = \frac{2}{9}$

c) $R_3(x) = \frac{f'''(x_0)}{3!}(x - x_0)^3$

$f'''(x) = \frac{12(x+1) \cdot 1}{(x+1)^4} = \frac{2}{(x+1)^3}$

$R_3(x) = \frac{1}{3(\xi+1)^3} x^3$

$x_0 = 0$

ξ je mezi 0 a $\frac{1}{3}$

$R_3(\frac{1}{3}) = \frac{1}{3(0+1)^3} \frac{1}{3^3} = \frac{1}{3^4} = \frac{1}{81}$

$|R_3(\frac{1}{3})| \leq \frac{1}{81}$

(2.) $\ddot{x} + \dot{x} - 6x = 0$

a) $\lambda^2 + \lambda - 6 = 0$

$D = 1 - (-6) \cdot 4 = 25$

$\lambda_{1,2} = \frac{-1 \pm 5}{2} = \begin{cases} -3 \\ 2 \end{cases}$

$\lambda_1 = 2$

$\lambda_2 = -3$

- fundamentální systém

$\{ \varphi_1(t) = e^{2t};\ \varphi_2(t) = e^{-3t} \}$

- obecné řešení

$x(t) = C_1 e^{2t} + C_2 e^{-3t}$ $\quad t \in (-\infty, +\infty)$

$x(t) = C_1 e^{2t} + C_2 e^{-3t}$

$\dot{x}(t) = 2C_1 e^{2t} - 3C_2 e^{-3t}$

P.P.: $x(0) = 0$

$\dot{x}(0) = 5$

$0 = C_1 + C_2 \quad / \cdot (+3)$

$5 = 2C_1 - 3C_2$

$5 = +C_1 5 \qquad C_2 = -1$

$C_1 = 1$

$x(t) = e^{2t} - e^{-3t} \; ; t \in (-\infty, +\infty)$

b) $\ddot{x} + \dot{x} - 6x = 10e^{2t}$

$\boxed{\lambda_1 = 2}$! $\rightarrow x_p = A \cdot t \cdot e^{2t}$

$\lambda_2 = -3$

$x_p = A \cdot t \cdot e^{2t}$

$\dot{x}_p = Ae^{2t} + Ate^{2t} \cdot 2$

$\ddot{x}_p = 2Ae^{2t} + A2e^{2t} + 4Ate^{2t}$

$4Ae^{2t} + Ae^{2t} + 2Ate^{2t} - 6Ate^{2t} = 10e^{2t} - \cancel{4Ate^{2t}}$

$5Ae^{2t} = 10e^{2t}$

$A = 2$

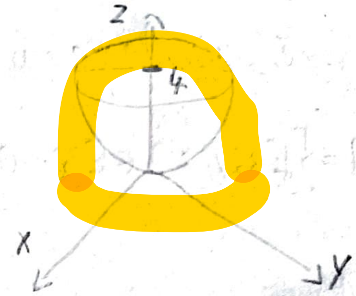
$x_p = Ate^{2t}$

$x_p = 2te^{2t}$

$x(t) = C_1 e^{2t} + C_2 e^{-3t} + 2te^{2t}$

$x(t) = C_1 e^{2t} + C_2 e^{-3t} + 2te^{2t} \; ; t \in (-\infty, +\infty)$

3. a) $Q = \{[x,y,z] \in E_3;\ z = 4 - x^2 - y^2\ ;\ z \geq 0\}$



$$\begin{aligned} z &= 4 - x^2 - y^2 \\ z &= x^2 + y^2 \\ \hline x^2 + y^2 &= 4 - x^2 - y^2 \end{aligned}$$

$$\begin{aligned} z &= 4 - x^2 - y^2 \\ z &\geq 0 \\ \hline x^2 + y^2 &\leq 4 \end{aligned}$$

$2(x^2+y^2) = 4$

$x^2 + y^2 = 2 \rightarrow r = 2$

$u = x$

$v = y$

$P(u,v) = [u, v, 4 - u^2 - v^2]$ $\qquad z = 4 - x^2 - y^2 = 4 - u^2 - v^2$

$P_u = [1, 0, -2u]$

$P_v = [0, 1, -2v]$

$$\vec{n} = P_u \times P_v = \begin{vmatrix} i & j & k \\ 1 & 0 & -2u \\ 0 & 1 & -2v \end{vmatrix} = +k + 2ui + 2vj = (+2u, +2v, 1)$$

Tok $\vec{f} = (x, y, 0)$

$$\iint_Q \vec{f}(x,y,z)\, d\vec{p} = \iint_B f(P(u,v))(P_u \times P_v)\, du\, dv =$$

$$= \iint_B (u, v, 0)(2u, 2v, 1)\, du\, dv = \iint_B (2u^2 + 2v^2)\, du\, dv = \left| \begin{array}{l} u = r \cdot \cos\varphi \\ v = r \cdot \sin\varphi \\ du\, dv = r\, dr\, d\varphi \\ 0 \leq r \leq 2 \\ 0 \leq \varphi \leq 2\pi \end{array} \right| =$$

$$= \int_0^{2\pi}\int_0^2 2r^3\, dr\, d\varphi = 4\pi \frac{2^4}{4} = \underline{16\pi}$$

b) vně orientovaná (+)

$M = \{[x,y,z] \in E_3 : 0 \le z \le 4 - x^2 - y^2\}$

$$+\iint_Q \vec{f}\, d\vec{p} = +\iiint_{\text{int } Q} \operatorname{div} \vec{f}\, dx\,dy\,dz$$

$\vec{f} = (x, y, 0) \quad \rightarrow \vec{f} = (U, V, W)$

$$\operatorname{div} \vec{f} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 1 + 1 + 0 = 2$$

$$\iiint_{0 \le z \le 4 - x^2 - y^2} 2\, dx\,dy\,dz = \left| \begin{array}{l|l} x = r \cdot \cos\varphi & 0 \le z \le 4 - x^2 - y^2 \\ y = r \cdot \sin\varphi & 0 \le z \le 4 - (r^2\cos^2\varphi + r^2\sin^2\varphi) \\ z = z & 0 \le z \le 4 - r^2 \\ dx\,dy\,dz = r\,dr\,d\varphi\,dz & 0 \le r \le 2 \qquad 0 \le \varphi \le 2\pi \end{array} \right| =$$

$$= \int_0^2 \int_0^{2\pi} \int_0^{4-r^2} 2r\, dr\,d\varphi\,dz = 2 \cdot 2\pi \int_0^2 r \cdot \Big[z\Big]_0^{4-r^2} dr = 4\pi \int_0^2 4r - r^3\, dr =$$

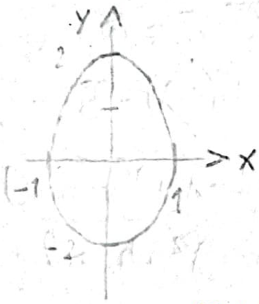
$$= 4\pi \left[4\frac{r^2}{2} - \frac{r^4}{4}\right]_0^2 = 4\pi\left(2 \cdot 2^2 - \frac{16}{4}\right) = \underline{16\pi}$$

2012/2013

① –

② – kladně orientovaná křivka $C=\{[x,y]\in E_2;\ x^2+\frac{y^2}{4}=1\}$

– elipsa s $a=1$
$b=2$



kladně orientovaná

a) $\oint_C \vec{f}\,d\vec{s} = \oplus \iint_{int\,C}\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy = \iint_{int\,C} 2\,dxdy = \begin{vmatrix} x=1\cdot r\cdot\cos\varphi \\ y=2\cdot r\cdot\cos\varphi \\ J=1\cdot 2\cdot r=2r \\ dxdy=2r\,dr\,d\varphi \\ r\to 0\le r\le 1 \\ 0\le\varphi\le 2\pi \end{vmatrix} =$

$\vec{f}=(\overset{P}{3x-y},\overset{Q}{x})$

$$=\int_0^{2\pi}\int_0^1 2\cdot 2r\,dr\,d\varphi = 8\pi\cdot\left[\frac{r^2}{2}\right]_0^1 = \underline{4\pi}$$

$\frac{\partial Q}{\partial x}=1 \qquad \frac{\partial P}{\partial y}=-1$

b) $\oint_C \vec{f}\,d\vec{s} = \int_C \vec{f}\,d\vec{s}$

– parametrizace

$x=1\cdot\cos t \qquad t\in(0,2\pi> \qquad P_1(t)=[\cos t;\ 2\sin t]$

$y=2\cdot\sin t \qquad\qquad \dot{P}(t)=[-\sin t;\ 2\cos t]$

$\vec{f}=(3x-y;\ x)=(3\cos t-2\sin t;\ \cos t)$

$$\oint_C \vec{f}\,d\vec{s} = \int_0^{2\pi}(3\cos t-2\sin t;\ \cos t)(-\sin t,\ 2\cos t)\,dt =$$

$$=\int_0^{2\pi} 2-3\cos t\sin t\,dt = \begin{vmatrix} x=\sin t \\ dx=\cos t\,dt \\ dt=\frac{1}{\cos t}dx \end{vmatrix} = \int_0^{2\pi}2\,dt-3\int_0^{2\pi}\cos t\cdot x\cdot\frac{dx}{\cos t} = \int_0^{2\pi}2\,dt-$$

$$3\cdot\left[\frac{x^2}{2}\right]_0^{2\pi} = 4\pi-3\cdot\left[\frac{\sin^2 t}{2}\right]_0^{2\pi} = \underline{4\pi}$$

(3.) $\dot{X} = \begin{pmatrix} -1 & -1 \\ 5 & 1 \end{pmatrix} X$

a) $A = \begin{pmatrix} -1 & -1 \\ 5 & 1 \end{pmatrix}$ $\quad \det(A-\lambda E) = 0 \Rightarrow \begin{vmatrix} -1-\lambda & -1 \\ 5 & 1-\lambda \end{vmatrix} = (-1-\lambda)(1-\lambda) + 5 = 0$

$-1+\lambda-\lambda+\lambda^2+5=0$

$\lambda^2+4=0$

$\underline{\lambda_{1,2} = \pm 2i}$

$\begin{pmatrix} -1+2i & -1 \\ 5 & 1+2i \end{pmatrix} \ldots\ldots$

2021/2022

(3.) $\ddot{x} + 2\dot{x} + 5x = 2\sin(2t)$

$\lambda^2 + \lambda \cdot 2 + 5 = 0$

$D = 4 - 4(5) = -16$

$\lambda_{1,2} = \frac{-2 \pm 4i}{2} = <$ $\quad \frac{-2+4i}{2} = -1+2i$, $\quad \frac{-2-4i}{2} = -1-2i$

$\lambda_1 = 2i - 1$ $\qquad \lambda = \alpha \pm \beta i$

$\lambda_2 = -2i - 1$ $\qquad \Rightarrow \alpha = -1, \beta = 2$

$x(t) = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$

– fundamentální systém

$\{\varphi_1(t) = e^{-t}\cos 2t;\ \varphi_2(t) = e^{-t}\sin 2t\}$

(4.) $\vec{f} = (-y, x, z)$

$M = \{[x,y,z] \in E_3;\ x^2+y^2+4 \leq z \leq 8\}$

$\operatorname{div} \vec{f} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0+0+1$ dovnitř $\rightarrow \ominus$

$-\iint_Q \vec{f}\, d\vec{p} = -\iiint_{\text{int } Q} 1\, dx\,dy\,dz = - \left| \begin{matrix} x = r\cdot\cos\varphi \\ y = r\sin\varphi \\ z = z \\ dx\,dy\,dz = r\,dr\,d\varphi\,dz \end{matrix} \quad \begin{matrix} 0 \leq r \leq 2 \\ 0 \leq \varphi \leq 2\pi \\ z:\ 4 + r^2 \leq z \leq 8 \end{matrix} \right| =$

$x^2+y^2+4 \leq z \leq 8$

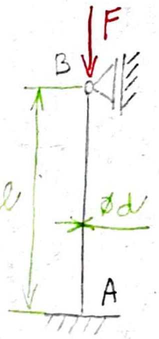
$$= -\int_0^2\int_0^{2\pi}\int_{4+r^2}^{8} r\,dr\,d\varphi\,dz = -2\pi\int_0^2 [z]_{4+r^2}^{8}\cdot r\,dr = -2\pi\int_0^2 r(4-r^2)\,dr = -2\pi\left[2r^2-\frac{r^4}{4}\right]_0^2 =$$

$$= \underline{-8\pi}$$

(5.) $\dot{x} = x - \frac{1}{y} \longrightarrow y \neq 0 \longrightarrow G_1 = \{[x,y] \in E_2;\ y>0\}$

$\dot{y} = 2x - y + 1 \qquad\qquad\qquad\qquad G_2 = \{[x,y] \in E_2;\ y<0\}$

PRUŽNOST + PEVNOST 2020/2021



D: $\sigma_K = R_e = 260\,N\,mm^{-2}$ $\quad E = 2\cdot 10^5\,N\,mm^{-2}$ $\quad d = 12\,mm$

$\sigma_u = R_u = 200\,N\,mm^{-2}$ $\quad l = 600\,mm$ $\quad k_E = 3$

U: $F^E_{Kr};\ \sigma_{Kr}$ a $F_D;\ \sigma_D$ — případ vzpěru III $\to n = 2$

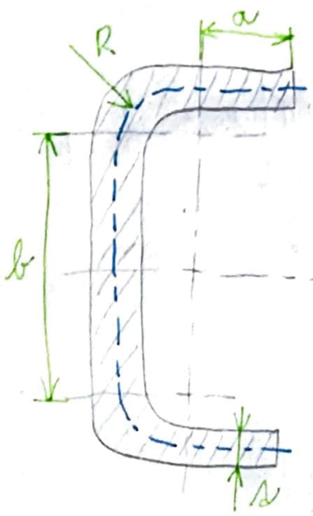
$$J_z = \frac{\pi d^4}{64} = \frac{\pi\cdot 12^4}{64} = 1017{,}88\,mm^4$$

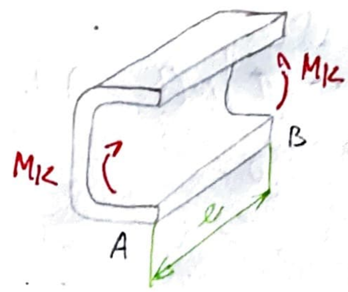
$$F^E_{KR} = n\,\frac{\pi^2 E J_{z\,min}}{l^2} = 2\cdot\frac{\pi^2\cdot 2\cdot 10^5\cdot 1017{,}88}{600^2} = \underline{11\,162{,}26\,N}$$

$$\sigma^E_{Kr} = \frac{F_{Kr}}{A} = \frac{4\cdot F_{Kr}}{\pi d^2} = \frac{4\cdot 11162{,}26}{\pi\cdot 12^2} = \underline{98{,}7\,N\,mm^{-2}} < \sigma_u = 200\,N\,mm^{-2}$$ — platí Euler

$$F_D = \frac{F^E_{Kr}}{k_E} = \frac{11\,162{,}62}{3} = \underline{3720{,}75\,N}$$

$$\sigma_D = \frac{F_D}{A} = \frac{4F_D}{\pi d^2} = \frac{4\cdot 3720{,}75}{\pi\cdot 12^2} = \underline{32{,}9\,N\,mm^{-2}}$$





D: $h = 130\,mm$, $s = 4\,mm$, $G = 0{,}8 \cdot 10^5\,MPa$

$l = 1200\,mm$ $R \cong 15\,mm$, $M_K = 2 \cdot 10^4\,Nmm$

$a = 25\,mm$, $b = 33\,mm$

U: τ_{max}, φ_{A-B} - vzájemné natočení konců

$$J_K = \frac{1}{3} h s^3 = \frac{1}{3} 130 \cdot 4^3 = \underline{2773{,}3\,mm^4}$$

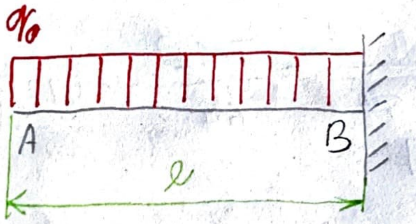
-polotovar-pás

$$W_K = \frac{1}{3} h s^2 = \frac{1}{3} 130 \cdot 4^2 = \underline{693{,}3\,mm^3}$$

$$\tau_{max} = \frac{M_K}{W_K} = \frac{2 \cdot 10^4}{693{,}3} = \underline{28{,}85\,Nmm^{-2}}$$

$$\varphi_{A-B} = \frac{M_K \cdot l}{G \cdot J_K} = \frac{2 \cdot 10^4 \cdot 1200}{0{,}8 \cdot 10^5 \cdot 2773{,}3} = \underline{0{,}108\,rad = 6{,}19°}$$

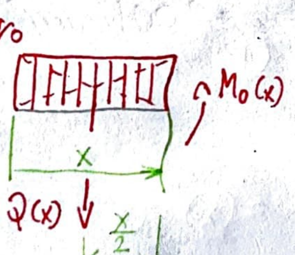
2019/2020



D: l, $E \cdot J_2 =$ konst, q_0

U: pomocí Bernoulliho rovnice určit natočení uprostřed nosníku $\varphi(x = \frac{l}{2})$

$M_0(x)$:



$$M_0(x) + Q(x) \cdot \frac{x}{2} = 0$$

$$M_0(x) = -\frac{x}{2} \cdot q_0 \cdot x = -\frac{x^2}{2} q_0$$

$$\boxed{\begin{aligned} v' &= \varphi \\ v'' &= M \end{aligned}}$$

$$v''(x) = -\frac{M_0(x)}{EJ_2} = \frac{q_0}{EJ_2} \frac{x^2}{2}$$

$$v'(x) = \frac{q_0}{EJ_2} \frac{x^3}{6} + C_1$$

$$v(x) = \frac{q_0}{EJ_2} \frac{x^4}{24} + C_1 x + C_2$$

O.P.: $v'(l) = \varphi(l) = 0$

$v(l) = 0$

$$C_1 = -\frac{q_0}{EJ_2} \frac{l^3}{6}$$

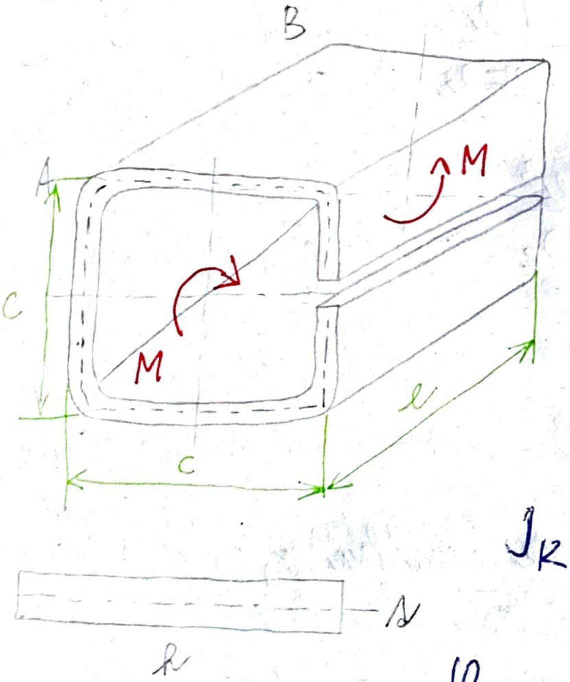
$$C_2 = -\frac{q_0}{EJ_2} \frac{l^4}{24} - C_1 \cdot l$$

$$C_2 = -\frac{q_0 l^4}{EJ_z}\cdot\frac{1}{24} + \frac{q_0}{EJ_z}\frac{l^4}{6} = \frac{q_0 l^4}{EJ_z}\left(-\frac{1}{24}+\frac{1}{6}\right) = \frac{q_0 l^4}{EJ_z}\,\frac{-1+4}{24} = \frac{q_0 l^4}{EJ_z}\,\frac{1}{8}$$

$$v(x) = \frac{q_0}{EJ_z}\frac{x^4}{24} - \frac{q_0 l^3}{6EJ_z}x + \frac{q_0 l^4}{EJ_z 8}$$

$$v\left(\frac{l}{2}\right) = \frac{q_0}{EJ_z}\left(\frac{1}{24}\frac{l^4}{16} - \frac{1}{6}l^3\frac{l}{2} + \frac{l^4}{8}\right) = \frac{q_0 l^4}{EJ_z}\,\frac{1-32+48}{384} = \frac{q_0 l^4}{EJ_z}\,\frac{17}{384}$$ -> průhyb

$$v'\left(\frac{l}{2}\right) = \frac{q_0}{EJ_z}\left(\frac{1}{6}\frac{l^3}{2^3} - \frac{l^3}{6}\right) = \frac{q_0 l^3}{EJ_z}\,\frac{1-8}{48} = \frac{q_0 l^3}{EJ_z}\left(-\frac{7}{48}\right) = \varphi\left(\frac{l}{2}\right)$$ -> natočení



- otevřený profil čtverce
- tloušťka stěny s
- délka střednice c

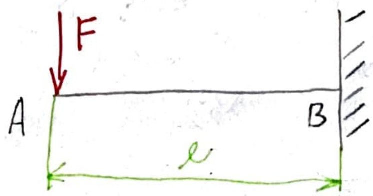
D: $l = 0{,}5\,m$ $\quad s = 3\,mm$ $\quad G = 0{,}8\cdot 10^5\,Nmm^{-2}$

$c = 30\,mm$ $\quad M = 1\cdot 10^4\,Nmm$

$$J_K = \frac{1}{3} l s^3 = \frac{1}{3}4\cdot c\cdot s^3 = \frac{1}{3}\cdot 4\cdot 30\cdot 3^3 = 1080\,mm^4$$

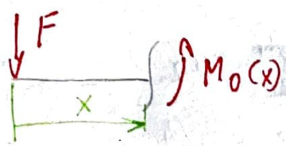
$$\varphi_{A-B} = \frac{M_K l}{G J_K} = \frac{1\cdot 10^4\cdot 500}{0{,}8\cdot 10^5\cdot 1080} = 0{,}058\,rad = 3{,}3^\circ$$

2018/20[redacted]19



U: natočení volného konce φ_A Bernoullim

D: EJ_z = konst., F



$M_0(x) = -F \cdot x$

$$v''(x) = -\frac{M_0(x)}{EJ_z} = \frac{F \cdot x}{EJ_z}$$

$$v'(x) = \frac{Fx^2}{2EJ_z} + C_1 \qquad \text{O.P.}: v(l) = 0$$

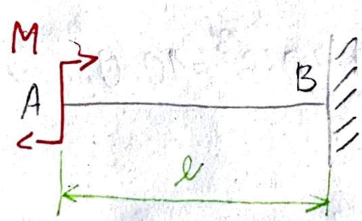
$$v'(l) = 0$$

$$v(x) = \frac{Fx^3}{6EJ_z} + C_1 x + C_2$$

$$0 = \frac{Fl^2}{2EJ_z} + C_1 \Rightarrow C_1 = -\frac{Fl^2}{2EJ_z}$$

$$C_2 = -\frac{Fl^3}{6EJ_z} + \frac{Fl^2}{2EJ_z} l = \frac{Fl^3}{EJ_z}\left(-\frac{1}{6} + \frac{1}{2}\right) = \frac{1}{3}\frac{Fl^3}{EJ_z}$$

$$\underline{\varphi_A} = \varphi(0) = v'(0) = \underline{-\frac{Fl^2}{2EJ_z}}$$



U: průhyb volného konce nosníku v_A

D: EJ_z = konst, M, l

$M_0(x) = M$

$$v''(x) = \frac{-M_0(x)}{EJ_z} = -\frac{M}{EJ_z}$$

$$v'(x) = -\frac{M \cdot x}{EJ_z} + C_1 \qquad \text{O.P.}: v'(l) = 0$$

$$v(l) = 0$$

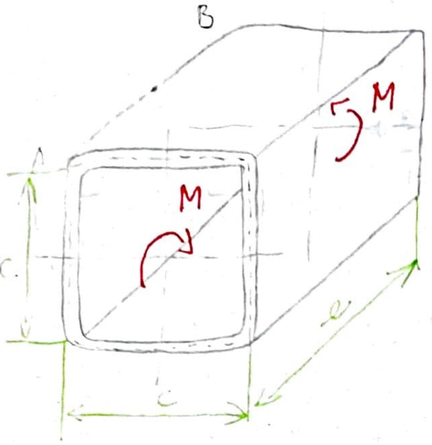
$$v(x) = \frac{-Mx^2}{2EJ_z} + C_1 x + C_2$$

$$C_1 = \frac{Ml}{EJ_z}$$

$$C_2 = \frac{Ml^2}{2EJ_z} - \frac{Ml^2}{EJ_z} = -\frac{1}{2}\frac{Ml^2}{EJ_z}$$

$$v(x) = -\frac{Mx^2}{2EJ_z} + \frac{Mlx}{EJ_z} - \frac{Ml^2}{2EJ_z}$$

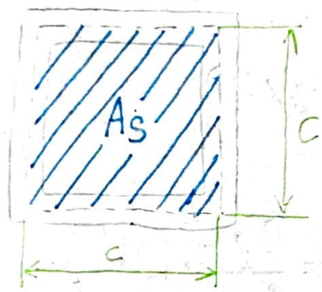
$$v(0) = v_A = \underline{-\frac{Ml^2}{2EJ_z}}$$



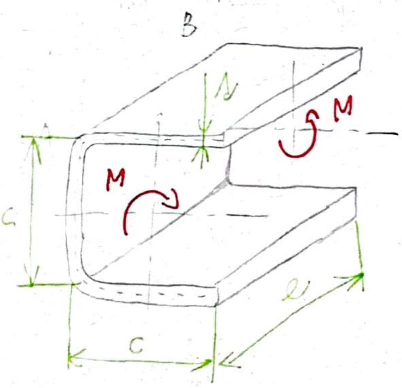
$D: l = 0{,}2\,m,\ c = 30\,mm,\ s = 2\,mm,\ M = 1{,}8\cdot 10^5\,Nmm$

$U: \tau_{max}$

$$\tau_{max} = \frac{M_k}{W_k} = \frac{M_k}{2\cdot A_s \cdot s_{min}} = \frac{M_k}{2\cdot c^2 \cdot s}$$



$$\underline{\tau_{max}} = \frac{1{,}8\cdot 10^5}{2\cdot 30^2 \cdot 2} = \underline{50\ MPa}$$



$D: l = 700\,mm \quad s = 2\,mm \quad G = 0{,}81\cdot 10^5\,Nmm$

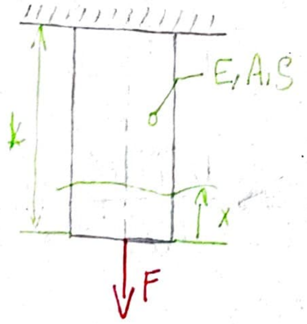
$c = 40\,mm \quad M = 3232\,Nmm$

$U: \varphi_{A-B}$

$$J_k = \frac{1}{3} l s^3 = \frac{1}{3} 3c \cdot s^3 = \frac{1}{3}\cdot 3\cdot 40\cdot 2^3 = 320\,mm^4$$

$$\underline{\varphi_{A-B}} = \frac{M_k \cdot l}{G J_k} = \frac{3232\cdot 700}{0{,}81\cdot 10^5 \cdot 320} = \underline{0{,}087\,rad = 5°}$$

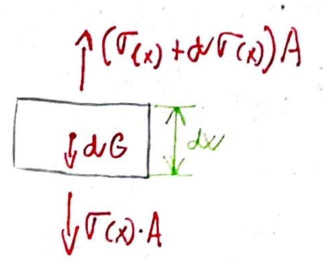
2017/2018



$D: F, E, A, \rho, L, g \quad U:$ pomocí DR $\sigma(x);\ u(x), \Delta L \quad \boxed{l = L}$

$$\sigma(x)\cdot A + dG - A(\sigma(x) + d\sigma(x)) = 0$$

$$dG - A\,d\sigma(x) = 0$$



$$g\cdot \rho A\,dx - A\,d\sigma(x) = 0$$

$$d\sigma(x) = g\rho\,dx$$

$$\sigma(x) = \rho g x + C_1$$

$O.P.: \sigma(0) = \frac{F}{A}$

$x = 0$

$$\Rightarrow C_1 = \frac{F}{A} \qquad \sigma(x) = \rho g x + \frac{F}{A}$$

- deformace elementu $\qquad \sigma = E \cdot \varepsilon$

$$\varepsilon(x) = \frac{dw(x)}{dx} \Rightarrow dw(x) = dx \cdot \frac{\sigma(x)}{E} = dx\,\frac{\rho g x + \frac{F}{A}}{E} = dx\left(\frac{\rho g}{E}x + \frac{F}{EA}\right)$$

$$w(x) = \frac{\rho g}{E}\frac{x^2}{2} + \frac{F}{EA}x + C_2 \qquad \text{O.P.: } w(L) = 0$$

$$0 = \frac{\rho g}{E}\frac{l^2}{2} + \frac{Fl}{EA} + C_2$$

$$C_2 = -\left(\frac{Fl}{EA} - \frac{\rho g}{E}\frac{l^2}{2}\right)$$

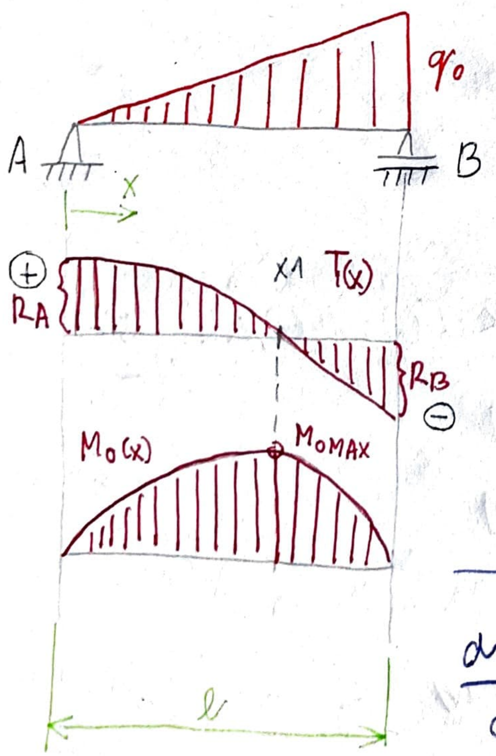
$$w(x) = \frac{\rho g}{E}\frac{x^2 - L^2}{2} + \frac{F}{EA}(x - L)$$

$$\Delta L = |w(0)| = \left|-\frac{\rho g L^2}{2E} - \frac{FL}{EA}\right| = \frac{\rho g L^2}{2E} + \frac{FL}{EA}$$

2016/2017

- jako 2020/2021

2015/2016



D: $l = 500\,mm$

$q_0 = 10\,N\,mm^{-1}$

$q(x) = q_0 \frac{x}{l}$

U: - pomocí Schwedlerovy věty průběhy $T(x)$ a $M_o(x)$

- M_{oMAX} a kde
- graficky průběhy $T(x)$ a $M_o(x)$

Schwedlerova věta

$$\frac{dT(x)}{dx} = -q(x) \;; \quad \frac{dM_o(x)}{dx} = T(x) \Rightarrow \frac{d^2 M_o(x)}{dx^2} = -q(x)$$

$$\frac{dT(x)}{dx} = -q_0\frac{x}{l}$$

$$dT(x) = -\frac{q_0}{l}x\,dx$$

$$T(x) = -\frac{q_0}{l}\frac{x^2}{2} + C_1$$

$$\frac{dM_0(x)}{dx} = -\frac{q_0}{l}\frac{x^2}{2} + C_1$$

O.P.: $M_0(0)=0$
$M_0(l)=0$

$$dM_0(x) = \left(-\frac{q_0}{2l}x^2 + C_1\right)dx$$

$$M_0(x) = -\frac{q_0}{6l}x^3 + C_1 x + C_2$$

$C_2 = 0$

$$0 = -\frac{q_0 l^3}{6l} + C_1 \cdot l$$

$$C_1 = \frac{q_0 l}{6}$$

$$T(x) = -\frac{q_0 x^2}{2l} + \frac{q_0 l}{6} = q_0\left(\frac{l}{6} - \frac{x^2}{2l}\right)$$

$$M_0(x) = -\frac{q_0 x^3}{6l} + \frac{q_0}{6} l x = q_0\left(\frac{lx}{6} - \frac{x^3}{6l}\right)$$

Reakce: $R_A \rightarrow x=0$

$$T(0) = \frac{q_0 l}{6} = R_A \qquad T(l) = q_0\left(\frac{l}{6} - \frac{l}{2}\right) = -\frac{q_0 l}{3} = R_B$$

$$T(x_1)=0 \rightarrow 0 = q_0\left(\frac{l}{6} - \frac{x_1^2}{2l}\right)$$

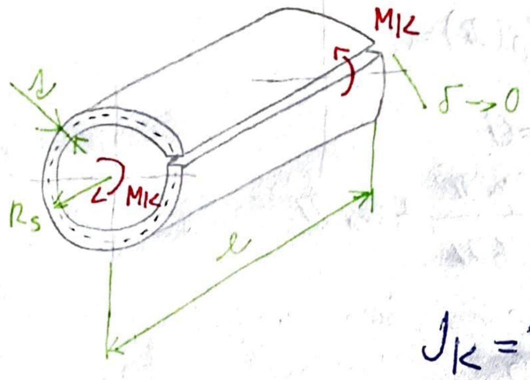
– tam kde je posouvající síla $T(x)=0$, tam je M_{OMAX}

$$0 = \frac{2l^2 - 6x_1^2}{12l}$$

$$6x_1^2 = 2l^2$$

$$x_1 = \sqrt{\frac{l^2}{3}} = \frac{l}{\sqrt{3}}$$

$$M_{OMAX}\left(\frac{l}{\sqrt{3}}\right) = q_0\left(\frac{l}{6}\frac{l}{\sqrt{3}} - \frac{1}{6l}\frac{l^3}{(\sqrt{3})^3}\right) = q_0\frac{l^2}{9\sqrt{3}}$$



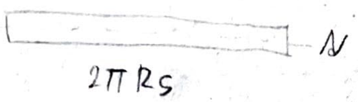
D: $R_S = 100\,mm$; $s = 3\,mm$; $l = 1200\,mm$

$G = 0{,}81 \cdot 10^5\,N\,mm^{-2}$ $\tau_D = 60\,N\,mm^{-2}$

U: M_{KD}

φ_{A-B}

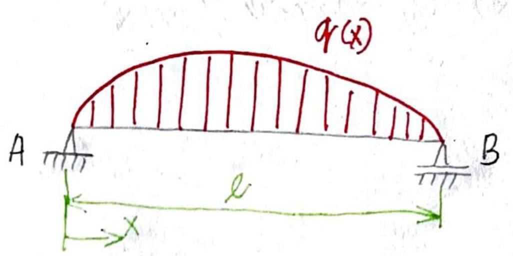
$$J_K = \frac{1}{3} 2\pi R_S\, s^3 = \frac{1}{3} 2\pi \cdot 100 \cdot 3^3 = 5654{,}9\,mm^4$$



$$\underline{M_{KD}} = W_K \cdot \tau_D = \frac{J_K}{s_{max}} \tau_D = \frac{5654{,}9}{3} 60 = \underline{113\,097\,Nmm}$$

$$G \cdot \vartheta_D = \frac{M_K}{J_K} = \frac{\tau_D}{s} \qquad \vartheta_D = \frac{\tau_D}{G \cdot s} = \frac{60}{0{,}81 \cdot 10^5 \cdot 3} = 0{,}247 \cdot 10^{-3}\,mm^{-1}$$

$$\underline{\varphi_{A-B} = \vartheta_D \cdot l} = 0{,}247 \cdot 10^{-3} \cdot 1200 = \underline{0{,}3\,rad \cong 17°}$$



D: $l = 500\,mm$

$q_0 = 10\,N \cdot mm^{-1}$

$q(x) = q_0 \cdot \sin\left(\frac{\pi}{l} x\right)$

U: Schwedlerkou $T(x)$ a $M_O(x)$

polohu a velikost $M_O(x)$

grafický průběh $T(x)$, $M_O(x)$

$$\frac{dT(x)}{dx} = -q(x) \qquad \frac{dM_O(x)}{dx} = T(x)$$

$$\int \sin\left(\frac{\pi}{l} x\right) dx = \left| \begin{array}{l} s = \frac{\pi}{l} x \\ ds = \frac{\pi}{l} dx \\ dx = \frac{l}{\pi} ds \end{array} \right| =$$

$$= \frac{l}{\pi} \int \sin s\, ds = \frac{l}{\pi} \left[-\cos s \right] =$$

$$= \frac{l}{\pi} \left(-\cos\left(\frac{\pi}{l} x\right) \right)$$

$$\frac{dT(x)}{dx} = -q_0 \sin\left(\frac{\pi}{l} x\right)$$

$$T(x) = q_0 \frac{l}{\pi} \cos\left(\frac{\pi}{l} x\right) + C_1$$

$$\frac{dM_O(x)}{dx} = q_0 \frac{l}{\pi} \cos\left(\frac{\pi}{l} x\right) + C_1$$

$$M_O(x) = q_0 \frac{l^2}{\pi^2} \sin\left(\frac{\pi}{l} x\right) + C_1 x + C_2$$

$O.P.:\ M(0)=0 \rightarrow C_2=0$

$M(l)=0 \rightarrow C_1=0$

$T(x)=q_0\frac{l}{\pi}\cos\left(\frac{\pi}{l}x\right)$ Reakce: $R_A=T(0)=q_0\frac{l}{\pi}$

$M_o(x)=q_0\frac{l^2}{\pi^2}\sin\left(\frac{\pi}{l}x\right)$ $R_B=T(l)=-q_0\frac{l}{\pi}$

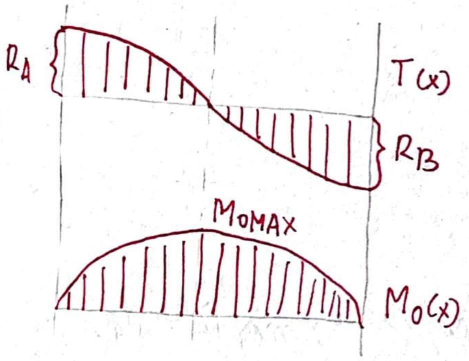
- poloha M_{OMAX} – tam, kde $T(x)=0$. V případě symetrického zatížení je to v polovině

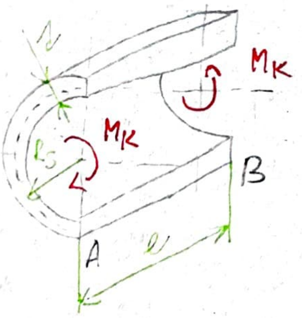
$$0=q_0\frac{l}{\pi}\cos\left(\frac{\pi}{l}x\right)$$

$$0=\cos\left(\frac{\pi}{l}x\right)$$

$$\Rightarrow x=\frac{1}{2}l=\frac{l}{2}$$

$$M_{OMAX}=q_0\frac{l^2}{\pi^2}\sin\left(\frac{\pi}{l}\cdot\frac{l}{2}\right)=q_0\frac{l^2}{\pi^2}$$



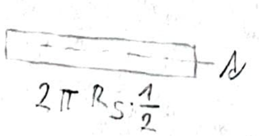


D: $R_S=100\,mm$, $s=3\,mm$, $l=1200\,mm$, $G=0{,}81\cdot10^5\,Nmm^{-2}$

$\tau_D=60\,Nmm^{-2}$

U: M_{KD}, φ_{A-B}

$$J_K=\frac{1}{3}Ls^3=\frac{1}{3}\pi R_S\cdot s^3=\frac{1}{3}\pi\cdot100\cdot3^3=2827{,}43\,mm^4$$

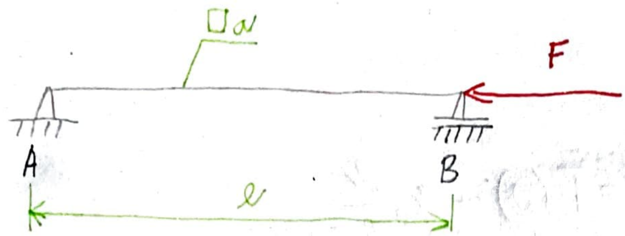


$$M_{KD}=\frac{J_K}{s_{max}}\tau_D=\frac{2827{,}43}{3}\cdot60=56\ 548{,}4\,Nmm$$

$$\vartheta_D=\frac{M_K}{J_K\cdot G}=\frac{J_K\cdot\tau_D}{J_K\cdot G\cdot s}=\frac{60}{0{,}81\cdot10^5\cdot3}=2{,}47\cdot10^{-4}\,mm^{-1}$$

$$\varphi_{A-B}=\vartheta_D\cdot l=2{,}47\cdot10^{-4}\cdot1200=0{,}3\,rad=17^\circ$$

2014/2015



D: $\sigma_k = R_e = 240\,MPa$ $E = 2\cdot10^5\,MPa$

$\sigma_u = R_u = 180\,MPa$ $l = 900\,mm$

$a = 16\,mm$ $k_\varepsilon = 4$

– případ 2. uložení $\Rightarrow n = 1$

U: $F_{kr}^{\varepsilon}, \sigma_{kr}, F_D, \sigma_D$

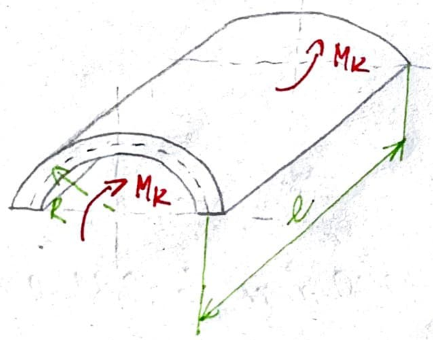
$J_{z\,min} = \frac{1}{12} a^4$

$$F_{kr}^{\varepsilon} = n \cdot \frac{\pi^2 E J_{z\,min}}{l^2} = n \frac{\pi^2 E a^4}{12 l^2} = \frac{\pi^2 \cdot 2\cdot10^5 \cdot 16^4}{12\cdot 900^2} = 13\,308{,}9\ N$$

$$\sigma_{kr}^{\varepsilon} = \frac{F_{kr}^{\varepsilon}}{A} = \frac{F_{kr}^{\varepsilon}}{a^2} = \frac{13\,308{,}9}{16^2} = 52\ MPa \leq \sigma_u$$ – platí Euler

$$F_D = \frac{F_{kr}^{\varepsilon}}{k_\varepsilon} = \frac{13\,308{,}9}{4} = 3\,327{,}2\ N$$

$$\sigma_D = \frac{F_D}{A} = \frac{3327{,}2}{16^2} = 13\ MPa$$



– pás $h \times l$

D: $h = 157\,mm$, $l = 600\,mm$, $s = 4\,mm$, $R \cong 50\,mm$

$G = 0{,}26\cdot10^5\,MPa$; $\tau_D = 30\,MPa$

U: M_{KD}, φ_{A-B}

$$J_K = \frac{1}{3} h s^3 = \frac{1}{3} 157 \cdot 4^3 = 3349{,}3\,mm^4$$

$$M_{KD} = W_K \cdot \tau_D = \frac{J_K}{s_{max}} \tau_D = \frac{3349{,}3}{4}\, 30 = 25\,119{,}75\ Nmm$$

$$\varphi_{A-B} = \frac{M_K \cdot l}{G \cdot J_K} = \frac{600 \cdot 25\,119{,}75}{0{,}26\cdot10^5 \cdot 3349{,}3} = 0{,}173\,rad = 9{,}9^\circ$$

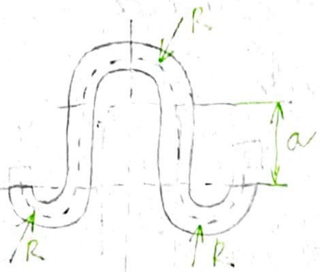
jinak

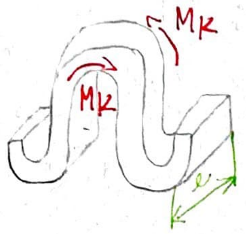
$$\vartheta_D = \frac{M_K}{J_K \cdot G} = \frac{J_K \cdot \tau_D}{s_{max}\, J_K \cdot G} = \frac{\tau_D}{s_{max} \cdot G} = \frac{30}{4 \cdot 0{,}26\cdot10^5} = 2{,}88\cdot10^{-4}\ mm^{-1}$$

$$\varphi_{A-B} = \vartheta_D \cdot l = 2{,}88\cdot10^{-4} \cdot 600 = 0{,}173\,rad$$

2013/2014

– 1. příklad jako 2020/2021





– pás $h \times l$ o tloušťce t

D: $h = 134\,mm$ $t = 3\,mm$ $a = 20\,mm$

$l = 120\,mm$ $R \cong 10\,mm$ $M_k = 2{,}5 \cdot 10^4\,Nmm$

$G = 0{,}81 \cdot 10^5\,MPa$

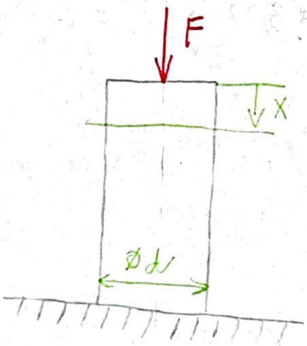
U: τ_{max}, φ_{A-B}

$$J_k = \frac{1}{3} h t^3 = \frac{1}{3} 134 \cdot 3^3 = 1206\,mm^4$$

$$\tau_{MAX} = \frac{M_k}{J_k} t_{max} = \frac{2{,}5 \cdot 10^4 \cdot 3}{1206} = 62{,}19\,MPa$$

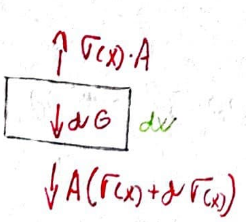
$$\varphi_{A-B} = \frac{M_k \cdot l}{G \cdot J_k} = \frac{2{,}5 \cdot 10^4 \cdot 1200}{0{,}81 \cdot 10^5 \cdot 1206} = 0{,}3\,rad = 17{,}6°$$

2012/2013



D: F, ρ, E, g, h, d

U: rovnice rovnováhy, průběh napětí, σ_{max}



$$\sigma(x)A - dG - A(\sigma(x) + d\sigma(x)) = 0$$

$$A\,d\sigma(x) = -\,g \cdot \rho \cdot A \cdot dx$$

$$d\sigma(x) = -\rho g\,dx$$

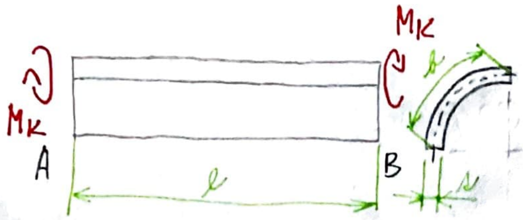
$$\sigma(x) = -\rho g x + C_1$$

O.P.: $\sigma(0) = -\frac{F}{A}$ TLAK

$$C_1 = -\frac{F}{A}$$

$$\sigma(x) = -\rho g x - \frac{F}{A}$$

$$\sigma_{max} = |\sigma(h)| = \rho g h + \frac{F}{A}$$



D: $s = 3\,mm$, $b = 60\,mm$, $l = 700\,mm$,

$G = 0{,}81 \cdot 10^5\,MPa$; $M_k = 15\,000\,Nmm$

– plech o rozměrech $b \times l$ a s

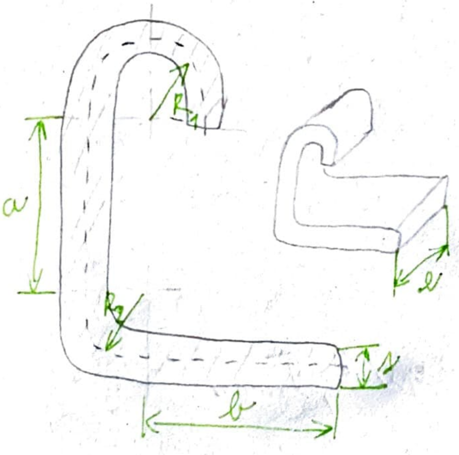
U: τ_{max}; φ_{A-B}

$$J_k = \frac{1}{3} b s^3 = \frac{1}{3} 60 \cdot 3^3 = 540\,mm^4$$

$$\tau_{max} = \frac{M_k}{W_k} = \frac{M_k}{\frac{J_k}{s}} = \frac{3 \cdot 15\,000}{540} = 83{,}3\,MPa$$

$$\varphi_{A-B} = \frac{M_k \cdot l}{G \cdot J_k} = \frac{15\,000 \cdot 700}{0{,}81 \cdot 10^5 \cdot 540} = 0{,}24\,rad = 13{,}75°$$

2021/2022



D: $G = 0{,}8 \cdot 10^5\,MPa$ $\quad b = 200\,mm$

$\tau_k = 80\,MPa$ $\quad s = 3\,mm$

$k_k = 2$ $\quad L = 100 \cdot s = 300\,mm$

$s = 3\,mm$ $\quad a = 70$

$R_1 = 15\,mm$ $\quad b = 52$

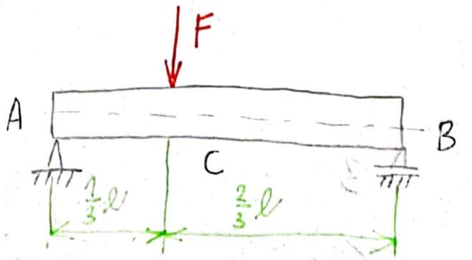
$R_2 = 20\,mm$

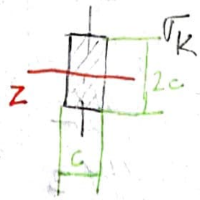
U: W_k, M_{kD}, φ_D při M_{kD}

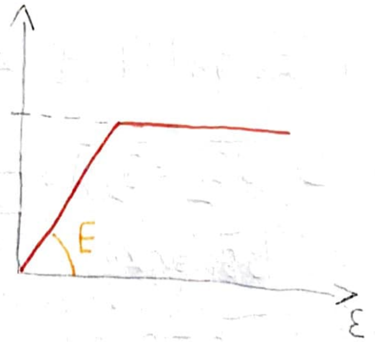
$$W_k = \frac{J_k}{s_{max}} = \frac{\frac{1}{3} b s^3}{s} = \frac{1}{3} b s^2 \qquad J_k = \frac{1}{3} b s^3 = \frac{1}{3} 200 \cdot 3^3 = 1800\,mm^4$$

$$M_{kD} = W_k \cdot \tau_D = \frac{1}{3} b s^2 \cdot \frac{\tau_k}{k} = \frac{1}{3} 200 \cdot 3^2 \cdot \frac{80}{2} = 24\,000\,Nmm$$

$$\varphi_D = \frac{M_{kD} \cdot l}{G \cdot J_k} = \frac{24\,000 \cdot 300}{0{,}8 \cdot 10^5 \cdot 1800} = 0{,}05\,rad$$







D: $\sigma_K = 200\ N\,mm^{-2}$

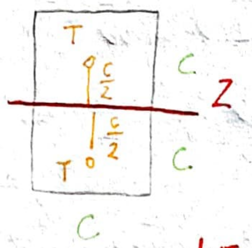
$l = 300\ mm$

$c = 30\ mm$

a) $W_{oz\,pl.}$ k ose z

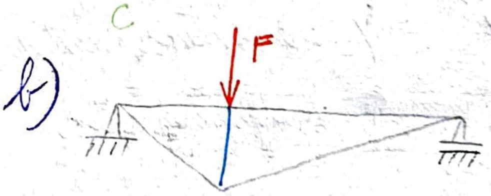
$W_0^{pl} = \sum S_{on\,pl.} = \sum$ obsah · vzdálenost těžiště od neutrální osy on

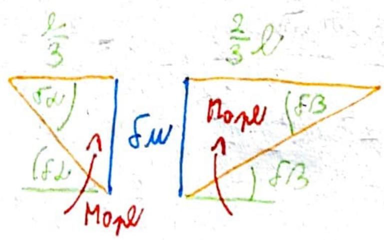
↑ suma stat. momentů k on



$$\underline{W_{oz\,pl.}} = 2 \cdot S_{on.\,pl} = 2 \cdot \left(c \cdot c \cdot \frac{1}{2} c\right) = \underline{c^3}$$

b)





$$\delta w = \frac{l}{3}\delta\alpha \rightarrow \delta\alpha = \frac{3}{l}\delta w \qquad W_{opl} = c^3 = 30^3$$

$$\delta w = \frac{2}{3} l\,\delta\beta \rightarrow \delta\beta = \frac{3}{2l}\delta w \qquad W_{oel} = \frac{1}{6} bh^2 = \frac{1}{6} \cdot c \cdot (2c)^2 =$$

$$F_{mez} \cdot \delta w = M_{opl}\,\delta\alpha + M_{opl}\,\delta\beta$$

$$F_{mez} \cdot \delta w = M_{opl}\left(\frac{3}{l}\delta w + \frac{3}{2l}\delta w\right)$$

$$F_{mez} = M_{opl}\,\frac{9}{2l}$$

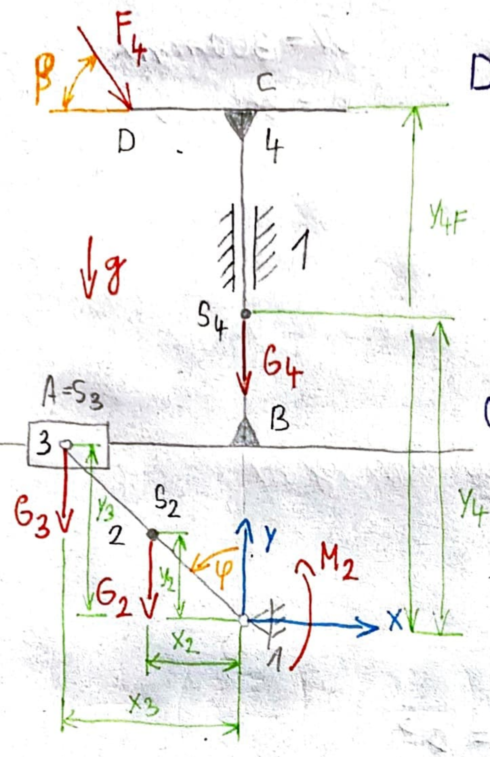
$$F_{mez} = \frac{9}{2l} \cdot \sigma_K \cdot W_{opl}$$

$$F_{el} = \frac{9}{2l}\,\sigma_K \cdot W_{oel}$$

$$F_{el} = \frac{9}{2l}\,\sigma_K \cdot \frac{2}{3} c^3 = \frac{9}{2 \cdot 300} \cdot 200 \cdot \frac{2}{3}\, 30^3$$

$$\underline{F_{el} = 54\,000\ N}$$

MECHANIKA 2020/2021



D: $\overline{OS_2} = \overline{S_2A} = \frac{r}{2}$ m_2 I_{2S2} β

$\overline{BS_4} = r$ m_3 I_{3S3} g

$\overline{S_4C} = c$ m_4 I_{4S4}

$\overline{CD} = d$ M_2

F_4

U: obecný tvar LR II. druhu a význam symbolů

kinetická energie mechanismu

zobecněná síla

popsat obecný způsob výpočtu LR II. druhu

sestavit vlastní pohybovou rovnici

1. $$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}}\right) - \frac{\partial E_k}{\partial q} = Q$$

E_k… kinetická energie

t… čas

q… zobecněná souřadnice

$\dot{q}$… zobecněná rychlost

Q… zobecněná síla

2. $$E_k = \frac{1}{2} I_{2O}\omega_2^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} m_4 v_4^2 \qquad I_{2O} = I_{2S2} + m_2\frac{r^2}{4}$$

$\omega_2 = \dot{\varphi}_2 = \dot{\varphi}$

$v_3 = \omega_2 \cdot r = \dot{\varphi}\, r$

$y_4 = r.\cos\varphi + r$

$v_4 = \dot{y}_4 = -r.\dot{\varphi}.\sin\varphi$

$$E_k = \frac{1}{2}\left(\left(I_{2S2} + m_2\frac{r^2}{4}\right) + m_3 r^2 + m_4 r^2 \sin^2\varphi\right)\dot{\varphi}^2$$

$x_3 = -r\sin\varphi$ $y_3 = r.\cos\varphi$ $v_3 = \sqrt{\dot{x}_3^2 + \dot{y}_3^2} = r.\dot{\varphi}$

$\dot{x}_3 = -r.\dot{\varphi}\cos\varphi$ $\dot{y}_3 = -r.\sin\varphi$

3. $$Q\tilde{\omega}_2 = M_2\tilde{\omega}_2 + m_2 g\tilde{v}_2\sin\varphi + m_3 g\tilde{v}_3\sin\varphi - m_4 g\tilde{v}_4 - F_4\sin\beta\,\tilde{v}_4$$

$\tilde{v}_2 = \frac{r}{2}\cdot\tilde{\omega}_2$

$\tilde{v}_3 = r\,\tilde{\omega}_2$

$\tilde{v}_4 = -r\,\tilde{\omega}_2\sin\varphi$

$$Q = M_2 + \left(\frac{m_2}{2} + m_3 + m_4\right) g r\sin\varphi + F_4\sin\beta\, r\sin\varphi$$

3. jinak

X složka je 0, nelze se do tohoto směru rozkbovat

$$Q\delta\varphi = M_2\delta\varphi - G_2\delta y_2 - G_3\delta y_3 - G_4\delta y_4 - F_4\cdot\sin\beta\,\delta y_{4F}$$

$y_2 = \frac{r}{2}\cos\varphi$ $\quad y_3 = r\cdot\cos\varphi$ $\quad y_4 = r\cdot\cos\varphi + r$ $\quad y_{4F} = r\cdot\cos\varphi + r + c$

$\delta y_2 = -\frac{r}{2}\sin\varphi\,\delta\varphi$ $\quad \delta y_3 = -r\sin\varphi\cdot\delta\varphi$ $\quad \delta y_4 = -r\cdot\sin\varphi\cdot\delta\varphi$ $\quad \delta y_{4F} = -r\cdot\sin\varphi\cdot\delta\varphi$

$$Q\delta\varphi = M_2\delta\varphi + G_2\frac{r}{2}\sin\varphi\,\delta\varphi + G_3\,r\sin\varphi\,\delta\varphi + G_4\,r\sin\varphi\,\delta\varphi + F_4\,r\sin\varphi\sin\beta\,\delta\varphi$$

$$Q = M_2 + \left(\frac{m_2}{2} + m_3 + m_4\right)g\sin\varphi\,r + F_4\,r\sin\varphi\sin\beta$$

4. $q = \varphi$ $\quad \dot{q} = \dot{\varphi}$

$$\frac{\partial E_k}{\partial\dot{\varphi}} = \dot{\varphi}\left(I_{2S_2} + m_2\frac{r^2}{4} + m_3 r^2 + m_4 r^2\sin^2\varphi\right)$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial\dot{\varphi}}\right) = \ddot{\varphi}\left(I_{2S2} + m_2\frac{r^2}{4} + m_3 r^2 + m_4 r^2\sin^2\varphi\right) + 2m_4 r^2\sin\varphi\dot{\varphi}\cdot\cos\varphi\dot{\varphi}$$

$$(\sin^2\varphi)' = (\sin\varphi\cdot\sin\varphi)' = (\cos\varphi\ \sin\varphi + \sin\varphi\cdot\cos\varphi\dot{\varphi}) = (2\sin\varphi\cos\varphi\cdot\dot{\varphi})$$ – pomůcka → derivace součinu

$$\frac{\partial E_k}{\partial\varphi} = \dot{\varphi}^2\cdot\frac{1}{2}m_4 r^2\, 2\cdot\sin\varphi\cdot\cos\varphi = m_4 r^2\sin\varphi\cos\varphi\,\dot{\varphi}^2$$

– pokud je derivace podle času, píše se i $\dot{\varphi}$, pokud jen podle q, $\dot{\varphi}$ se nepíše.

$$\ddot{\varphi}\left(I_{2S2} + m_2\frac{r^2}{4} + m_3 r^2 + m_4 r^2\sin^2\varphi\right) + m_4 r^2\sin\varphi\cos\varphi\,\dot{\varphi}^2 = M_2 + \left(\frac{m_2}{2} + m_3 + m_4\right)g\cdot r\cdot\sin\varphi + F_4\,r\sin\varphi\sin\beta$$

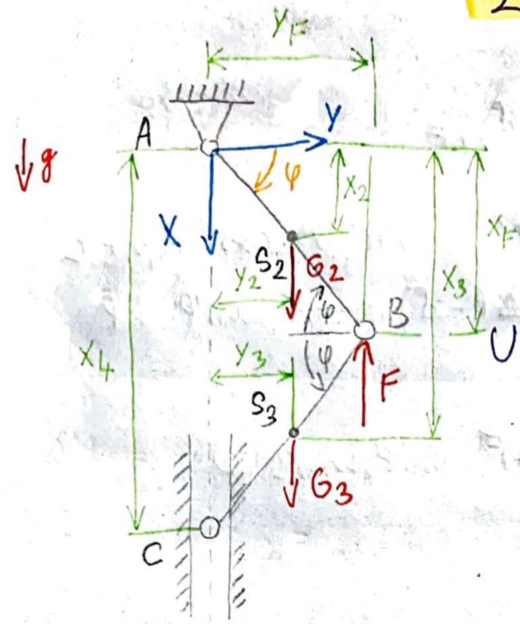
2019/2020

stejný

2018/2019

stejný

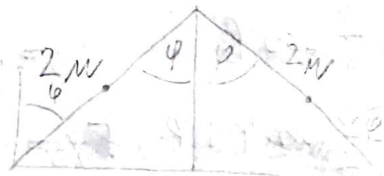
2017/2018



D: $\overline{AS_2} = \overline{S_2B} = \overline{BS_3} = \overline{S_3C} = r$

$m_2 \; I_{2S} \quad F, g$

$m_3 \; I_{3S}$



U: LR II obecný tvar

kin. energie

zobecněná síla

vlastní pohyb. rovnice

1. $\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}}\right) - \left(\frac{\partial E_k}{\partial q}\right) = Q$

t... čas

E_k... kin. energie

q... zobecněná souřadnice

$\dot{q}$... zobecněná rychlost

Q... zobecněné síly/a

2. $E_k = \frac{1}{2} I_{20} \, \omega_2^2 + \frac{1}{2} m_3 v_3^2 + \frac{1}{2} I_{3S} \, \omega_3^2$ $\qquad I_{20} = I_{3S} + m_2 r^2$

$\omega_2 = \dot{\varphi}$

$x_3 = 2r\sin\varphi + r \cdot \sin\varphi = 3r\sin\varphi \qquad y_3 = 2r \cdot \cos\varphi - r \cdot \cos\varphi = r \cdot \cos\varphi$

$\dot{x}_3 = 3r \cdot \cos\varphi \cdot \dot{\varphi} \qquad \dot{y}_3 = -r \cdot \sin\varphi \cdot \dot{\varphi}$

$v_3 = \sqrt{\dot{x}_3^2 + \dot{y}_3^2} \Rightarrow v_3^2 = 9r^2\cos^2\varphi \cdot \dot{\varphi}^2 + r^2\sin^2\varphi\,\dot{\varphi}^2 = r^2\dot{\varphi}^2(1 + 8\cos^2\varphi)$

$1 = \cos^2\varphi + \sin^2\varphi$

$\sin^2\varphi = 1 - \cos^2\varphi$

$\omega_3 = \dot{\varphi}$

$E_k = \frac{1}{2}\left[I_{2S} + m_2 r^2 + m_3 r^2 (1 + 8\cos^2\varphi) + I_{3S}\right]\dot{\varphi}^2$

(3.) $Q\delta\varphi = G_2\cdot\delta x_2 + G_3\delta x_3 - F\delta x_F$

$x_2 = r\cdot\sin\varphi$ $\quad x_3 = 3r\sin\varphi$ $\quad x_F = 2r\sin\varphi$

$\delta x_2 = r\cdot\cos\varphi\cdot\delta\varphi$ $\quad \dot{x}_3 = 3r\cos\varphi\cdot\delta\varphi$ $\quad \delta x_F = 2r\cos\varphi\cdot\delta\varphi$

$$Q\delta\varphi = m_2 g r\cos\varphi\delta\varphi + m_3 g\, 3r\cos\varphi\delta\varphi - F\,2r\cos\varphi\delta\varphi$$

$$Q = r\cdot\cos\varphi\left(m_2 g + 3m_3 g - 2F\right)$$

(4) $$\frac{\partial E_k}{\partial\dot\varphi} = \dot\varphi\left(I_{2S} + m_2 r^2 + m_3 r^2(1+8\cos^2\varphi) + I_{3S}\right)$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial\dot\varphi}\right) = \ddot\varphi\left(I_{2S} + m_2 r^2 + m_3 r^2(1+8\cos^2\varphi) + I_{3S}\right) + 16m_3 r^2\cos\varphi(-\sin\varphi)\cdot\dot\varphi^2$$

$$\frac{\partial E_k}{\partial\varphi} = \dot\varphi^2\cdot\frac{1}{2}m_3 r^2\, 16\cos\varphi\cdot(-\sin\varphi)$$

$$\ddot\varphi\left(I_{2S} + m_2 r^2 + m_3 r^2(1+8\cos^2\varphi) + I_{3S}\right) + 16m_3 r^2\cos\varphi(-\sin\varphi)\dot\varphi^2 + \dot\varphi^2 8m_3 r^2\cos\varphi\sin\varphi = r\cdot\cos\varphi\left(m_2 g + 3m_3 g - 2F\right)$$

$$\ddot\varphi\left(I_{2S} + m_2 r^2 + m_3 r^2(1+8\cdot\cos^2\varphi) + I_{3S}\right) - 8m_3 r^2\cos\varphi\sin\varphi\,\dot\varphi^2 = r\cdot\cos\varphi\left(G_2 + 3G_3 - 2F\right)$$

2016/2017

jako 2020/2021

2015/2016

jako 2020/2021 ale otočené o 90°

2014/2015

jako 2017/2018

2013/2014

jako 2020/2021, ale zrcadlově otočené

ČMS 2018/2019

1. pro čelní ozubené soukolí s přímými zuby a parametry

$m=2$ $\quad z_1=20$ $\quad x_1=+0{,}1$ $\qquad$ U: roztečné kružnice obou kol d_1, d_2

$i=3{,}8$ $\quad z_2=76$ $\quad x_2=-0{,}1$ $\qquad$ hlavové kružnice obou kol d_{a1}, d_{a2}

osovou vzdálenost soukolí pro souběr bez boční vůle a_w

$$\varnothing d_1 = m \cdot z_1 = 2\cdot 20 = 40$$

$$\varnothing d_2 = m \cdot z_2 = 2\cdot 76 = 152$$

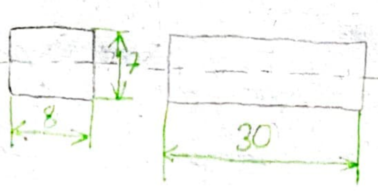
$$\varnothing d_{a1} = d_1 + 2m(h_{a1}^* + x_1) = d_1 + 2m(1+x_1) = 40+2\cdot 2(1+0{,}1) = 44{,}4$$

$$\varnothing d_{a2} = d_2 + 2m(h_{a2}^* + x_2) = d_2 + 2m(1+x_2) = 152+2\cdot 2(1-0{,}1) = 155{,}6$$

$$a_w = a = \frac{1}{2}(d_1+d_2) = \frac{1}{2}(40+152) = 96$$

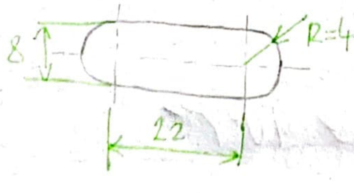
2. mezi hřídelem a nábojem o ⌀ hřídele $d=28$ mm přenáší točivý moment o velikosti $M=105$ Nm těsné pero rozměry $8\times 7\times 30$. Určete velikost kontaktního tlaku na boku pera. Určete velikost smykového napětí pro kontrolu pera na střih

$$F\cdot r = M \Rightarrow F\frac{d}{2} = M \Rightarrow F = \frac{2M}{d} = \frac{2\cdot 105}{0{,}028} = 7500\,N$$ – přenášená síla



$S_1 = 77 = 22\cdot 3{,}5\ mm = 77\ mm^2$ – kontaktní plocha boku pera na tlačení

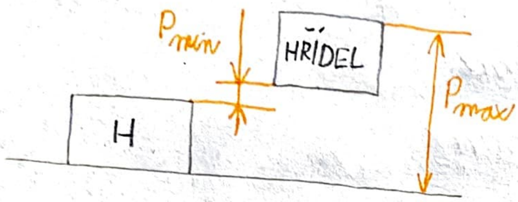
$S_2 = 22\cdot 8 = 176\ mm^2$ – střižná plocha



$$p = \frac{F}{S_1} = \frac{7500}{77} = 97{,}4\,MPa$$

$$\tau = \frac{F}{S_2} = \frac{7500}{176} = 42{,}6\,MPa$$

3. Nakreslete schéma tolerančních polí pro uložení díry a hřídele v soustavě ISO s přesahem. Vyznačte nejmenší a největší přesah. Pro uložení použijte soustavu jednotné díry.



1. $m=2$ $\quad z_1 = 20$ $\quad x_1 = +0{,}1$ $\quad \varnothing d_1 = m \cdot z_1 = 2 \cdot 20 = 40$

$i = 4{,}2$ $\quad z_2 = 84$ $\quad x_1 = -0{,}1$ $\quad \varnothing d_2 = m \cdot z_2 = 2 \cdot 84 = 168$

$$d_{a1} = d_1 + 2m(h^*_{a1} + x_1) = d_1 + 2m(1+x_1) = 40 + 2 \cdot 2 \cdot (1+0{,}1) = 44{,}4$$

$$d_{a2} = d_2 + 2m(h^*_{a2} + x_2) = d_2 + 2m(1+x_2) = 168 + 2 \cdot 2(1-0{,}1) = 171{,}6$$

$$a_w = a = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}(40+168) = 104$$

2. $d = 40\,mm$ $\quad M = F \cdot r \Rightarrow F = \frac{2M}{d} = \frac{2 \cdot 314}{0{,}04} = 15\,700\,N$

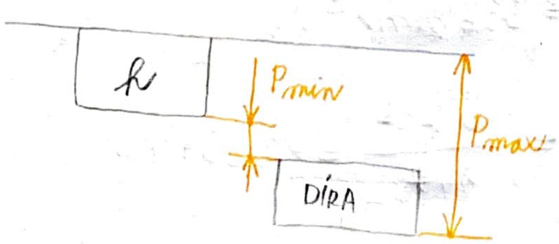
$M = 314\,Nm$

pero $12 \times 8 \times 50$

$S_1 = 4 \cdot (50-6-6) = 152\,mm^2$ $\quad p = \frac{F}{S_1} = \frac{15700}{152} = 103{,}3\,MPa$

$S_2 = 12(50-6-6) = 456\,mm^2$ $\quad \tau = \frac{F}{S_2} = \frac{15\,700}{456} = 34{,}4\,MPa$

3. – pro uložení použijte soustavu jednotného hřídele



2019/2020

stejné jako 2018/2019, ale jiné hodnoty

(1.) a)

$m = 2\,mm$ $\quad x_1 = 0,1$

$i = 3,79$ $\quad x_2 = -0,1$

$z_1 = 19$

U: $z_2, d_1, d_2, d_{a1}, a_w$

$\varnothing d_1 = m \cdot z_1 = 2 \cdot 19 = 38\,mm$

$\varnothing d_2 = m \cdot z_2 = 2 \cdot 72 = 144\,mm$

$i = \frac{z_2}{z_1} \Rightarrow z_2 = z_1 \cdot i$

$z_2 = 19 \cdot 3,79$

$z_2 = 72$

$\varnothing d_{a1} = d_1 + m \cdot 2(h_{a0}^* + x_1) = d_1 + 2m(1 + x_1)$

$\varnothing d_{a1} = 38 + 2 \cdot 2(1 + 0,1) = 42,4\,mm$

$a_w = a = \frac{1}{2}(d_1 + d_2) = 0,5(38 + 144) = 91\,mm$

(1) b)

$m = 2\,mm$ $\quad$ U: $z_2, d_1, d_2, d_{a1}, a_w$

$i = 3,62$

$z_1 = 21$ $\quad z_2 = z_1 \cdot i$ $\quad \varnothing d_1 = m \cdot z_1 = 2 \cdot 21 = 42\,mm$

$x_1 = 0,1$ $\quad z_2 = 21 \cdot 3,62$ $\quad \varnothing d_2 = m \cdot z_2 = 2 \cdot 76 = 152\,mm$

$x_2 = -0,1$ $\quad z_2 = 76$

$\varnothing d_{a1} = d_1 + 2m(h_a^* + x_1) = d_1 + 2m(1 + x_1)$

$\varnothing d_{a1} = 42 + 2 \cdot 2(1 + 0,1) = 46,4\,mm$

$a_w = a = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}(42 + 152) = 97\,mm$

(2) $\varnothing d = 28\,mm$

pero $8 \times 7 \times 50$

$M = 150\,Nm$

$S_1 = 3,5 \cdot (50 - 4 - 4) = 147\,mm^2$

$S_2 = 8 \cdot (50 - 4 - 4) = 336\,mm^2$

$F = \frac{M}{\frac{d}{2}} = \frac{2 \cdot 150}{0,028} = 10\,714\,N$

$p = \frac{F}{S_1} = \frac{10\,714}{147} = 72,88\,MPa$

$\tau = \frac{F}{S_2} = \frac{10\,714}{336} = 31,89\,MPa$

② $\varnothing d = 32\,mm$

pero $10 \times 8 \times 50$

$M = 225\,Nm$

$$F = \frac{2M}{d} = \frac{2 \cdot 225}{0{,}032} = 14\,062{,}5\,N \doteq 14\,063\,N$$

$$S_1 = 4(50-5-5) = 160\,mm^2$$

$$S_2 = 10(50-5-5) = 400\,mm^2$$

$$p = \frac{F}{S_1} = \frac{14\,063}{160} = 87{,}89\,MPa$$

$$\tau = \frac{F}{S_2} = \frac{14\,063}{400} = 35{,}16\,MPa$$

③ stejné

2020/2021

①

$m = 2\,mm$ $\quad i = 3{,}99$ $\quad$ U: $z_2, d_1, d_{a1}, d_f, a_w$

$z_1 = 19$

$x_1 = x_2 = 0$

$$z_2 = i \cdot z_1$$

$$z_2 = 3{,}99 \cdot 19 = 75{,}81 \doteq 76$$

$$d_1 = m \cdot z_1 = 2 \cdot 19 = 38\,mm$$

$$d_{a1} = d_1 + 2m(x_1+1) = 38 + 2 \cdot 2(1)$$

$$d_{a1} = 42\,mm$$

$$d_{f1} = d_1 - 2{,}5m$$

$$d_{f1} = 38 - 2{,}5 \cdot 2 = 33\,mm$$

$$a_w = a = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}(38 + 152) = 95\,mm$$

$$d_2 = m \cdot z_2 = 2 \cdot 76 = 152\,mm$$

② $d = 32\,mm$

$M = 160\,Nm$

pero $10 \times 8 \times 50$

$$F = \frac{2M}{d} = \frac{M}{r} = \frac{2 \cdot 160}{0{,}032} = 10\,000\,N$$

$$S_1 = 4(50-5-5) = 160\,mm^2$$

$$S_2 = 10(50-5-5) = 400\,mm^2$$

2017/2018

(1.) $m = 2$ $\quad$ U: $a, a_w, d_2, d_{a2}, d_{f2}$

$i = 3{,}9$

$z_1 = 20$

$x_1 = 0{,}1$

$x_2 = -0{,}1$

součinitel výšky hlavy zubu $= 1$ $= h_a^*$

součinitel hlavové vůle $= 0{,}25$ $= c^*$

$z_2 = i \cdot z_1 = 3{,}9 \cdot 20 = 78$

$d_1 = m \cdot z_1 = 2 \cdot 20 = 40\,mm$

$d_2 = m \cdot z_2 = 2 \cdot 78 = 156\,mm$

$a = a_w = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}(40 + 156) = 98\,mm$

$d_{a2} = d_2 + 2m(h_{a2}^* + x_2) = 156 + 2 \cdot 2(1 - 0{,}1) = 159{,}6\,mm$

$d_{f2} = d_2 - 2m(h_{a2}^* + c^*) + 2xm = 156 - 2 \cdot 2(1 + 0{,}25) - 2 \cdot 0{,}1 \cdot 2 = 150{,}6\,mm$

(2.) přiřadit těsné pero

$\varnothing d = 25\,mm$ $\quad$ U: délku pera minimální $p = 50\,MPa$

$M = 90\,Nm$ $\quad$ τ pro tuto délku pera

$p = 50\,MPa$

$F = \frac{2M}{d} = \frac{2 \cdot 90}{0{,}025} = 7200\,N$

- podle $\varnothing d$ je pero 8×7

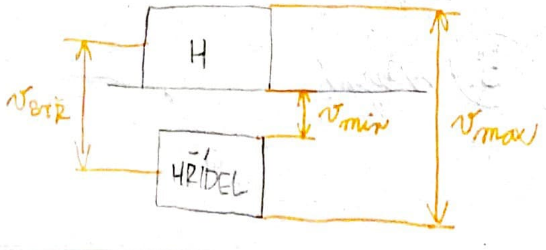
$S_1 = 3{,}5 \cdot l_a$

$p = \frac{F}{S_1} \Rightarrow S_1 = \frac{F}{p}$

$3{,}5 \cdot l_a = \frac{F}{p} \Rightarrow l_a = \frac{F}{3{,}5 \cdot p} = \frac{7200}{3{,}5 \cdot 50} = 41{,}1\,mm$

$S_2 = l_a \cdot b = 41{,}1 \cdot 8$ $\quad$ $\tau = \frac{F}{S_2} = \frac{7200}{41{,}1 \cdot 8} = 21{,}9\,MPa$

(3). soustava ISO s vůlí – jednotná díra



1. $U: a, a_w, d_2, d_{a2}, d_{f2}, (h_a^* = 1, c^* = 0{,}25)$

$m = 2\,mm$

$i = 4{,}1\,mm$

$z_1 = 20$

$x_1 = 0{,}1$

$x_2 = -0{,}1$

$$z_2 = i \cdot z_1 = 4{,}1 \cdot 20 = 82$$

$$d_1 = m \cdot z_1 = 2 \cdot 20 = 40\,mm$$

$$d_2 = m \cdot z_2 = 2 \cdot 82 = 164\,mm$$

$$a_w \doteq a = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}(40 + 164) = 102\,mm$$

$$d_{a2} = d_2 + 2m(h_{a2}^* + x_2) = d_2 + 2m(1 - 0{,}1) = 164 + 2 \cdot 2(0{,}9) = 167{,}6\,mm$$

$$d_{f2} = d_2 - 2m(h_{a2}^* + c_2) + 2xm = 164 - 2 \cdot 2(1 + 0{,}25) + 2 \cdot (-0{,}1) \cdot 2 = 158{,}6\,mm$$

2. $d = 35\,mm$ $\rightarrow$ pero 10×8

$M = 250\,Nm$

$p = 50\,MPa$

$$F = \frac{2M}{d} = \frac{2 \cdot 250}{0{,}035} = 14\,285{,}7\,N$$

$S_1 = 4 \cdot l_a$

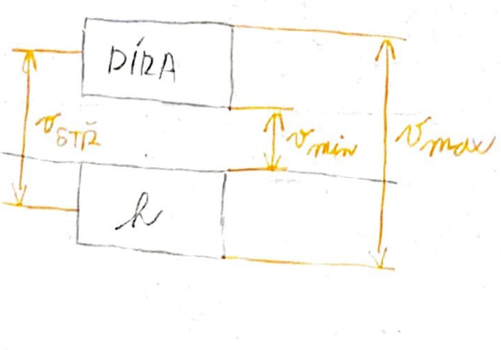
$S_2 = 10 \cdot l_a$

$$p = \frac{F}{S_1} = \frac{F}{4 \cdot l_a} \Rightarrow l_a = \frac{F}{4p}$$

$$l_a = \frac{14\,285{,}7}{4 \cdot 50} = 71{,}4\,mm$$

$$\tau = \frac{F}{S_2} = \frac{F}{10 \cdot l_a} = \frac{14\,285{,}7}{10 \cdot 71{,}4} = 20\,MPa$$

3. soustava ISO uložení - jednotný hřídel



2016/2017

1. a) $M_K = 92\,Nm$

$\tau_D = 30\,MPa$

$$\tau_D = \frac{M_K}{W_K} = \frac{M_K}{\frac{\pi d^3}{16}} \Rightarrow d = \sqrt[3]{\frac{16 M_K}{\pi \tau_D}} = \sqrt[3]{\frac{16 \cdot 92000}{\pi\, 30}}$$

$d = 25\,mm$

b) pro ⌀25 → pero 8 x 7

c) $p = 50\,MPa$

$$F = \frac{2M}{d} = \frac{2 \cdot 92000}{25} = 7360\,N$$

$S_1 = 3{,}5 \cdot l_a$

$$p = \frac{F}{S_1} = \frac{F}{3{,}5 \cdot l_a}$$

$$l_a = \frac{F}{3{,}5 \cdot p} = \frac{7360}{3{,}5 \cdot 50} = 42{,}1\,mm$$

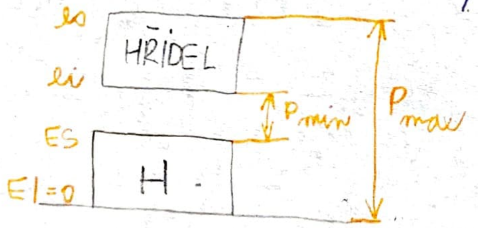
$l_{min} = l_a + boky = l_a + 4 + 4 = 50{,}1\,mm$

⇒ normalizovaná délka $l = 63\,mm$

d) $\tau_{DS} = 50\,MPa$ – kontrola vybraného pera – tzn. normalizovaného?

$$\tau = \frac{F}{S_2} = \frac{F}{8 \cdot (63-4-4)} = \frac{7360}{8 \cdot 55} = 16{,}7\,MPa$$

2. přesah ISO soustava, jednotná díra



2015/2016

A varianta

$U: a, a_w, d, d_a, d_f, d_b$

$m = 2$ $z_1 = 20$

$i = 2{,}91$ $x_1 = 0{,}1$

$h_a^* = 1$ $x_2 = -0{,}1$

$c^* = 0{,}25$ $\alpha = 20°$

$z_2 = i \cdot z_1 = 2{,}91 \cdot 20 = 58$

$d_1 = m \cdot z_1 = 2 \cdot 20 = 40\,mm$

$d_2 = m \cdot z_2 = 2 \cdot 58 = 116\,mm$

$a = a_w = \frac{1}{2}(d_1 + d_2) = 78\,mm$

$d_{a1} = d_1 + 2m(h_{a1}^* + x_1) = 40 + 2 \cdot 2(1 + 0{,}1) = 44{,}4\,mm$

$d_{a2} = d_2 + 2m(h_{a2}^* + x_2) = 116 + 2 \cdot 2(1 - 0{,}1) = 119{,}6\,mm$

$d_{f1} = d_1 - 2m(h_{a1}^* + c^*) + 2x_1 m = 40 - 2 \cdot 2(1 + 0{,}25) + 2 \cdot 0{,}1 \cdot 2 = 35{,}4\,mm$

$d_{f2} = d_2 - 2m(h_{a2}^* + c^*) + 2x_2 m = 116 - 2 \cdot 2(1 + 0{,}25) + 2 \cdot (-0{,}1) \cdot 2 = 110{,}6\,mm$

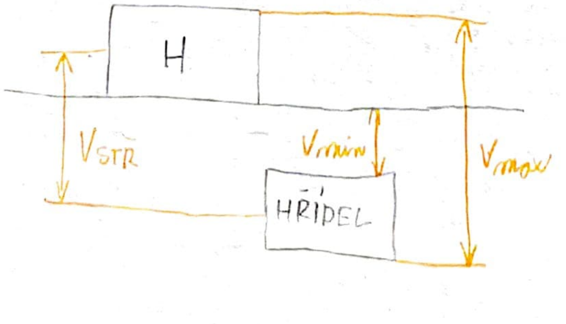
$d_{b1} = d_1 \cdot \cos\alpha = 40 \cdot \cos 20 = 37{,}588\,mm$

$d_{b2} = d_2 \cdot \cos\alpha = 116 \cdot \cos 20 = 109\,mm$

- velikost σ_D v stupém svaru namáhaný kolmou silou

$\sigma_{D\perp} = \alpha\perp \cdot \sigma_D$ $\sigma_D = \frac{\sigma_{Ke}}{k}$

- soustava ISO s vůlí - jednotná díra



B varianta

$m = 2$ U: $a, d_1, d_2, d_a, d_f, d_b$

$i = 3{,}11$ $h_a^* = 1$

$z_1 = 20$ $c^* = 0{,}25$

$x_1 = 0{,}1$ $\alpha = 20°$

$x_2 = -0{,}1$

$z_2 = i \cdot z_1 = 3{,}11 \cdot 20 = 62$

$d_1 = m \cdot z_1 = 2 \cdot 20 = 40$

$d_2 = m \cdot z_2 = 2 \cdot 62 = 124$

$a = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}(40 + 124)$

$a = 82\,mm$

$d_{a1} = d_1 + 2m(h_{a1}^* + x_1) = 40 + 2 \cdot 2(1 + 0{,}1) = 44{,}4\,mm$

$d_{a2} = d_2 + 2m(h_{a2}^* + x_2) = 124 + 2 \cdot 2(1 - 0{,}1) = 127{,}6\,mm$

$d_{f2} = d_2 - 2m(h_{a2}^* + c_2^*) + 2x_2 m = 124 - 2 \cdot 2(1 + 0{,}25) - 2 \cdot 0{,}1 \cdot 2 = 118{,}6\,mm$

$d_{f1} = d_1 - 2m(h_{a1}^* + c_1^*) + 2x_1 m = 40 - 2 \cdot 2(1 + 0{,}25) + 2 \cdot 0{,}1 \cdot 2 = 35{,}4\,mm$

$d_{b1} = d_1 \cdot \cos\alpha = 40 \cdot \cos 20 = 37{,}58\,mm$

$d_{b2} = d_2 \cdot \cos\alpha = 124 \cdot \cos 20 = 116{,}5\,mm$

- určení σ_D v tupém svaru namáhaného souběžnou silou

$\sigma_{D\parallel} = \alpha_\parallel \cdot \sigma_D$ $\sigma_D = \frac{\sigma_{Kt}}{k}$

- uložení v ISO s vůlí, jednotný hřídel